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Refining the Formulation of Geometric Optics as an Abelian U(1) Gauge Theory: Addressing Conceptual Inaccuracies and Enhancing Predictive Accuracy

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Abstract

The formulation of geometrical optics as an Abelian U(1) gauge theory was proposed in the study "Geometrical Optics as an Abelian U(1) Gauge Theory in a Vacuum Space-Time" (Indonesian Physical Review, Volume 7, Issue 1, January 2024). While innovative, this approach contains conceptual inaccuracies that must be addressed to ensure consistency with fundamental theoretical predictions, such as the straight-line propagation of light in homogeneous media or vacuum. This article examines these issues in depth and proposes a refined theoretical framework to correct and enhance the existing formulation.

Introduction

Geometric optics is one of the classical approaches in physics to describe the propagation of light, especially in the limit of very small wavelengths ($\lambda \rightarrow 0$). This approach uses the concept of light rays propagating through a medium with a certain refractive index, and is often used to understand the phenomena of refraction, reflection, and optical paths in various complex media [1][2][3]. In the electromagnetic framework, geometric optics can be derived from Maxwell's equations through the eikonal equation approach, which describes the path of light as a classical limit solution of electromagnetic waves [4][5][6][7]. This approach has also been extended to include anisotropic media and refractive index gradients using the gauge field [8][9][10].

Recently, research has attempted to reformulate geometric optics within the framework of Abelian U(1) gauge theory, as proposed in the article "Geometrical Optics as an Abelian U(1) Gauge Theory in a Vacuum Space-time" [11]. The article claims that the optical phase, refractive index, and propagation of light beams can be understood through gauge potentials, field strength tensors, and topological structures. This claim is mathematically interesting because it utilizes the formalism of gauge theory which is often used to explain particle physics and quantum field phenomena [12][13][14][15][16][17][18]. Previously, several studies have indeed demonstrated the application of gauge theory to light propagation in complex media, such as anisotropic media and metamaterials [19][20][21][22].

Nevertheless, the formulation contains conceptual inaccuracies that prevent it from making physical predictions, such as light propagating in straight lines in a medium with a homogeneous refractive index or in a vacuum. In the following analysis, these inaccuracies will be addressed, and the formulation of geometrical optics within the framework of an Abelian U(1) gauge theory will be corrected to ensure its physical accuracy.

Phase Formula

The article "*Geometrical Optics as an Abelian U(1) Gauge Theory in a Vacuum Space-time*" explores geometrical optics through the framework of Abelian U(1) gauge field theory [11]. The authors reformulate the eikonal equation as a gauge theory in a (3+1)-dimensional vacuum space-time using a weak-field approximation. Key equations include the gauge potential

$$\boldsymbol{A}_\mu = \boldsymbol{a}_\mu(\boldsymbol{r},t)e^{iq(\boldsymbol{r},t)} \quad (1)$$

and the field strength tensor

$$\boldsymbol{F}_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\mu \boldsymbol{A}_\nu \quad (2)$$

which describe the optical field in terms of the refractive index $n(r)$ and other variables. Numerical simulations reveal that the refractive index can be approximated as $n(r)$=1.0001 in vacuum space-time and that the weak magnetic field magnitude $|B|$=0.10452 T supports the weak-field approximation.

The vacuum space-time is interpreted as a weak-field limit, where electromagnetic fields are of very low intensity. The study introduces the phase $q(\boldsymbol{r},t)$, related to the refractive index, expressed as:

$$q(\boldsymbol{r},t) = X\left\{\int_{r_1}^{r_2} n(\boldsymbol{r})\, d^3r - ct\right\}, \quad (3)$$

where X is a constant. The refractive index is modeled as

$$n(r) = n_0\left(1 - \frac{ar^2}{2}\right), \quad (4)$$

showing that it decreases with increasing distance from the source. The authors also define the amplitude $\rho(\boldsymbol{r},t)$ and phase $q(\boldsymbol{r},t)$ in the scalar field

$$\phi(\boldsymbol{r},t) = \rho(\boldsymbol{r},t)e^{iq(\boldsymbol{r},t)}, \quad (5)$$

highlighting its topological and isotropic properties in vacuum.

Numerical results show the refractive index decreases radially, confirming vacuum characteristics. The study also finds the weak magnetic field magnitude $|B|$=0.10452 T, computed using

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}. \tag{6}$$

These findings reveal an innovative perspective connecting geometrical optics with gauge field theory, opening new avenues for exploring topological structures and weak-field conditions.

Correction of the Phase Formula

In this section, the phase equation (Equation (3)), which serves as the basis for proposing geometric optics as an Abelian U(1) gauge theory in a vacuum Space-time, will be investigated using fundamental concepts.

The visible light wave propagates through a vacuum with a velocity of [1][2][5]

$$\begin{aligned} c &= \lambda f \\ &= \frac{2\pi f}{\frac{2\pi}{\lambda}} \\ &= \frac{\omega}{k} \end{aligned} \tag{7}$$

where f is the frequency of the light, ω is the angular frequency, λ is the wavelength of the light, and k is the wave number of the light, along with

$$\begin{aligned} \omega &= \frac{2\pi}{T} \\ &= 2\pi f \end{aligned} \tag{8}$$

and

$$k = \frac{2\pi}{\lambda}. \tag{9}$$

When the light propagates through a medium, a change in the wavelength occurs, while the frequency remains constant. The wave speed of light in the medium will then become

$$\begin{aligned} v &= \lambda' f \\ &= \frac{2\pi f}{\frac{2\pi}{\lambda'}} \\ &= \frac{\omega}{k'} \end{aligned} \tag{10}$$

with

$$k' = \frac{2\pi}{\lambda'}. \tag{11}$$

Meanwhile, the refractive index is defined as [1][2][5]

$$n = \frac{c}{v}\ . \tag{12}$$

The combination of Equations (10) and (12), followed by rearrangement, will yield the equation

$$k' = \frac{\omega}{c} n\ . \tag{13}$$

The wave speed of light in the medium also satisfies the equation [1][2][5]

$$v = \frac{l}{t} \tag{14}$$

where l is the distance traveled by light within the medium and t is the time interval for light propagation through the medium.

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The optical path length or the distance effectively traveled by the light wave is defined as [1][2][5]

$$\Delta = ct. \tag{15}$$

The combination of Equations (12), (14), and (15) will yield the equation

$$\begin{aligned}\Delta &= ct \\ &= c\left(\frac{l}{v}\right) \\ &= nl.\end{aligned} \tag{16}$$

Multiplying Equation (10) by l and relating it to Equation (16), the resulting equation will be

$$\begin{aligned}k'l &= \frac{\omega}{c}nl \\ &= \frac{\omega}{c}\Delta\,.\end{aligned} \tag{17}$$

The phase in Equations (1) and (5) satisfies the equation [1][2][5]

$$\begin{aligned}q(\boldsymbol{r},t) &= \frac{2\pi}{\lambda'}l - \frac{2\pi}{T}t \\ &= k'l - \omega t.\end{aligned} \tag{18}$$

The combination of Equations (17) and (18) will yield the equation

$$\begin{aligned}q(\boldsymbol{r},t) &= k'l - \omega t \\ &= \frac{\omega}{c}\Delta - \omega t \\ &= \frac{\omega}{c}(\Delta - ct).\end{aligned} \tag{19}$$

When light travels along a path l defined by the position vectors r_1 and r_2, the optical path length will satisfy the equation

$$\Delta = \int_{r_1}^{r_2} n(\boldsymbol{r})dl. \tag{20}$$

The phase of the light wave therefore satisfies the combined Equations (19) and (20), expressed as:

$$q(\boldsymbol{r},t) = \frac{\omega}{c}\left(\int_{r_1}^{r_2} n(\boldsymbol{r})dl - ct\right). \tag{21}$$

Based on Equation (21), it can be concluded that Equation (3) contains a mathematical form that is not physically clear, namely: $\int_{r_1}^{r_2} n(\boldsymbol{r})\, d^3r$. The correct form of the integral should be: $\int_{r_1}^{r_2} n(\boldsymbol{r})\, dl$, which corresponds to the concept of optical path length [1][2][3][4].

Consequences of the Correction

Referring to Equation (21), Equation (3) is revised to become:

$$q(\boldsymbol{r},t) = X\left\{\int_{r_1}^{r_2} n(\boldsymbol{r})\, dl - ct\right\}. \tag{22}$$

Subsequently, it will be demonstrated that this revision provides an accurate approximation of light wave propagation in both homogeneous media and vacuum.

In Equation (22), one may select

$$q_1 = \int_{r_1}^{r_2} n(\boldsymbol{r})\, dl. \tag{23}$$

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The shortest wave propagation path from r_1 to r_2 at any time satisfies the condition:

$$\delta q_1 = 0. \tag{24}$$

It can be found the variation of q_1 as [23][24]

$$\delta q_1 = \delta \int_{r_1}^{r_2} n(\boldsymbol{r})\, dl$$
$$= \int_{r_1}^{r_2} n(\boldsymbol{r})\, \delta(dl) + \int_{r_1}^{r_2} \delta n(\boldsymbol{r})\, dl. \tag{25}$$

In Equation (25), there are two relations, namely:

$$\delta(dl) = \delta(\boldsymbol{\tau}.d\boldsymbol{r})$$
$$= \boldsymbol{\tau}.\delta(d\boldsymbol{r}) + \delta\boldsymbol{\tau}.d\boldsymbol{r}$$
$$= \boldsymbol{\tau}.\delta(d\boldsymbol{r}) + 0.d\boldsymbol{r}$$
$$= \boldsymbol{\tau}.\delta(d\boldsymbol{r}) \tag{26}$$

and

$$\delta n(\boldsymbol{r}) = \frac{\partial n}{\partial x}\delta x + \frac{\partial n}{\partial y}\delta y + \frac{\partial n}{\partial z}\delta z$$
$$= \left(\boldsymbol{i}\frac{\partial n}{\partial x} + \boldsymbol{j}\frac{\partial n}{\partial y} + \boldsymbol{k}\frac{\partial n}{\partial z}\right).(\boldsymbol{i}\delta x + \boldsymbol{j}\delta y + \boldsymbol{k}\delta z)$$
$$= \boldsymbol{\nabla} n.\delta\boldsymbol{r}. \tag{27}$$

By combining Equations (24), (25), (26) and (27), it can be obtained

$$\delta q_1 = \int_{r_1}^{r_2} n(\boldsymbol{r})\, \boldsymbol{\tau}.\mathrm{d}(\delta\boldsymbol{r}) + \int_{r_1}^{r_2} \delta\boldsymbol{r}.\boldsymbol{\nabla} n(\boldsymbol{r})\, dl$$
$$= \{n(\boldsymbol{r})\boldsymbol{\tau}.\delta\boldsymbol{r}\}_{\boldsymbol{r_1}}^{\boldsymbol{r_2}} - \int_{r_1}^{r_2} \delta\boldsymbol{r}.dn(\boldsymbol{r})\boldsymbol{\tau} + \int_{r_1}^{r_2} \boldsymbol{\nabla} n(\boldsymbol{r}).\delta\boldsymbol{r}dl$$
$$= 0 - \int_{r_1}^{r_2} \frac{dn(\boldsymbol{r})\boldsymbol{\tau}}{dl}.\delta\boldsymbol{r}\, dl + \int_{r_1}^{r_2} \boldsymbol{\nabla} n(\boldsymbol{r}).\delta\boldsymbol{r}dl$$
$$= \int_{r_1}^{r_2} \left\{\boldsymbol{\nabla} n(\boldsymbol{r}) - \frac{dn(\boldsymbol{r})\boldsymbol{\tau}}{dl}\right\}.\delta\boldsymbol{r}dl = 0 \tag{28}$$

where $\boldsymbol{\tau}$ is the vector of tangential units with respect to the direction of wave propagation. From Equation (28), it can be acquired the following relationship:

$$\boldsymbol{\nabla} n(\boldsymbol{r}) = \frac{dn(\boldsymbol{r})\boldsymbol{\tau}}{dl}. \tag{29}$$

Expanding Equation (29), it can be found the following Equation:

$$\boldsymbol{\nabla} n(\boldsymbol{r}) = n(\boldsymbol{r})\frac{d\boldsymbol{\tau}}{dl} + \boldsymbol{\tau}\frac{dn(\boldsymbol{r})}{dl}$$
$$= n(\boldsymbol{r})\frac{d\boldsymbol{\tau}}{dl} + \boldsymbol{\tau}\big(\boldsymbol{\tau}.\boldsymbol{\nabla} n(\boldsymbol{r})\big) \tag{30}$$

where

$$\frac{dn(\boldsymbol{r})}{dl} = \frac{\frac{\partial n(r)}{\partial r}dr}{dl}$$

$$= \frac{\partial n(\boldsymbol{r})}{\partial r}\frac{dr}{dl}$$
$$= \left(\boldsymbol{\tau}\frac{dr}{dl}\right).\left(\boldsymbol{\tau}\frac{\partial n(\boldsymbol{r})}{\partial r}\right)$$
$$= (\boldsymbol{\tau}|\boldsymbol{\tau}|).(\boldsymbol{\nabla}n(\boldsymbol{r}))$$
$$= \boldsymbol{\tau}.\boldsymbol{\nabla}n(\boldsymbol{r}). \tag{31}$$

Equation (30) can be rearranged so that it can be found the following equation:

$$\frac{d\boldsymbol{\tau}}{dl} = \frac{1}{n(\boldsymbol{r})}\{\boldsymbol{\nabla}n(\boldsymbol{r}) - \boldsymbol{\tau}(\boldsymbol{\tau}.\boldsymbol{\nabla}n(\boldsymbol{r}))\}. \tag{32}$$

Meanwhile, it is known that

$$\boldsymbol{\tau}.\boldsymbol{\tau} = 1 \tag{33}$$

and thus, it can be determined

$$\boldsymbol{\tau}.\frac{d\boldsymbol{\tau}}{dl} = 0. \tag{34}$$

It is also known that

$$\boldsymbol{\tau}.\boldsymbol{\eta} = 0 \tag{35}$$

where $\boldsymbol{\eta}$ is the vector of normal units (perpendicular to the direction) of wave propagation. When the left-hand and right-hand sides of Equation (34) are multiplied by the radius of curvature of the optical path R and connected to Equation (35), the resulting equation is obtained [24][25]:

$$\frac{d\boldsymbol{\tau}}{dl} = \frac{\boldsymbol{\eta}}{R}. \tag{36}$$

The combination of Equations (32) and (36) gives the following equation:

$$\frac{\boldsymbol{\eta}}{R} = \frac{1}{n(\boldsymbol{r})}\{\boldsymbol{\nabla}n(\boldsymbol{r}) - \boldsymbol{\tau}(\boldsymbol{\tau}.\boldsymbol{\nabla}n(\boldsymbol{r}))\}. \tag{37}$$

If Equation (37) is subjected to a dot operation with $\boldsymbol{\eta}$, it will give the equation

$$\frac{1}{R} = \frac{\boldsymbol{\eta}}{n(\boldsymbol{r})}.\boldsymbol{\nabla}n(\boldsymbol{r}). \tag{38}$$

Based on Equation (38), it can be deduced that the direction of wave propagation is influenced by variations in the refractive index. Specifically, as the refractive index increases, the wave undergoes a change in direction. Consequently, when a wave propagates in a homogeneous medium where $n(\boldsymbol{r})$ is constant, or in free space where $n(\boldsymbol{r})=1$, so $\boldsymbol{\nabla}n(\boldsymbol{r}) = 0$, the wave trajectory remains unaffected by refraction. In such cases, the bending radius R approaches infinity, indicating that the wave path is linear.

Conclusion

The phase formula in the proposed formulation of geometric optics as an Abelian U(1) gauge theory has been revised and validated, providing accurate predictions for the propagation of light in a medium with a homogeneous refractive index and in a vacuum.

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