Transformation of 3-D Jerk Chaotic System into Parallel Form

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***Abstract*—The paper deals with the developing of the methodological backgrounds for studying 3-D chaotic systems. Such backgrounds allow us to perform the coordinate transformation for 3-D nonlinear dynamical object from serial form into parallel one. Above-mentioned transformation is based on the partial fraction decomposition of the systems' feedforwards. Usage of the proposed approach is one of the ways of constructing a system with a chaotic dynamic and defining novel attractors. The approach has been proven by considering the example of modeling and simulating of third order chaotic system.**

Keywords—3-D jerk chaotic system; chaotic dynamic; coordinate transformation; partial fraction decomposition

I. INTRODUCTION

Many areas of mankind activities deal with dynamical systems which are sensitive to initial conditions [1]. One can find that various processes in meteorology, communication, robotic, chemistry, finance, sociology, medicine, and others are studied by using systems with a chaotic dynamic.

Since Lorenz discovered his 3-D chaotic models [2], one can mark that nonlinear third order dynamical systems are a very powerful tool for studying chaotic dynamic and performing chaos control [3-10]. There are a lot of various systems which can generate this type of oscillations [11-27].

These systems are different from each other by structure and parameters. That is why, it is very hard to find some common features, to predict systems characteristics and to discover their properties.

It would be preferred that dynamic chaotic system is represented by one state space before performing any comparison. Moreover, transformation into the parallel form is useful from computational, methodological and control viewpoints. Partial fraction decomposition ws used in this work to perform transformation of such kind.

The paper is organized as follows: at first, the transformation of the generalized 3-D chaotic system was considered as the main topic which dynamic is given in canonical state space. At second, observer's algorithm was offered for defining first and second derivatives of the system's outputs. Then, an example of transformation for 3-D jerk chaotic system and studying its dynamic was performed.

II. GENERALIZED 3-D JERK CHAOTIC SYSTEM'S MODEL

A. *Transformation of chaotic system's dynamic into counter-parallel form*

Generalized controllable 3-D jerk system dynamic was given as (1)

$$\dot{x}_1 = x_2; \dot{x}_2 = x_3; \dot{x}_3 = f(x_1, x_2, x_3) + m_3 u, \qquad (1)$$

where x_1, x_2, x_3 are state variables, u is a control effort, $f(x_1, x_2, x_3)$ is some function, m_3 is some coefficient.

Afterward, the function $f(x_1, x_2, x_3, u)$ was replaced with (2).

$$f(x_1, x_2, x_3) = g(x_1, x_2, x_3) + \sum_{i=1}^{3} a_i x_i, \qquad (2)$$

where a_i are some coefficients. it was assumed that coefficients a_i are nonzero coefficients and coefficient m_3 is positive one.

Coefficients a_i was offered to be defined in such a manner that in polynomial expressed in (3)

$$D(\lambda) = \lambda^3 + a_3\lambda^2 + a_2\lambda + a_1 = 0 \qquad (3)$$

has three different negative real eigenvalues $\lambda_1, \lambda_2, \lambda_3$. In that case, unknown function $g(x_1, x_2, x_3)$ was defined as (4).

$$g(x_1, x_2, x_3) = f(x_1, x_2, x_3) - a_1 x_1 - a_2 x_2 - a_3 x_3, \qquad (4)$$

where

$$a_1 = -\lambda_1\lambda_2\lambda_3; a_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3; \quad a_3 = -\lambda_1 - \lambda_2 - \lambda_3. \tag{5}$$

The above-given transformation has led to the rewritten of (1) into pseudo-affine form

$$\dot{x}_1 = x_2; \dot{x}_2 = x_3; \dot{x}_3 = g(x_1, x_2, x_3) + \sum_{i=1}^{3} a_i x_i + m_3 u . \tag{6}$$

Then, the equation in (6) has been transformed into operator form as (7).

$$sx_1 = x_2; sx_2 = x_3; sx_3 = g(x_1, x_2, x_3) + \sum_{i=1}^{3} a_i x_i + m_3 u , \tag{7}$$

where $s = d/dt$ is a derivative operator.

Block diagram of chaotic system (7) is shown in Fig. 1.

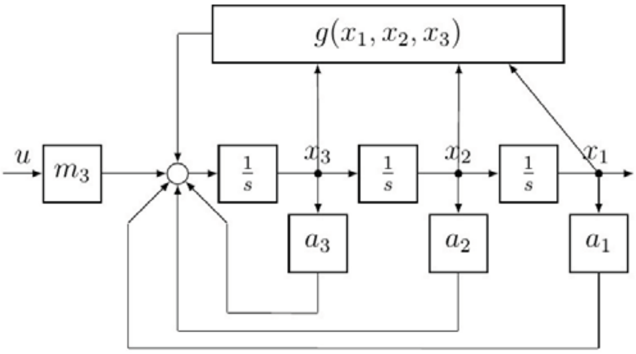

Fig. 1. Block diagram of generalized 3-D jerk chaotic system

Equations (7) can be rewritten as (8).

$$s^3 x_1 = \left[\sum_{i=1}^{3} a_i s^{i-1} x_1 + m_3 u\right] + \left[g\left(x_1, sx_1, s^2 x_1\right)\right]. \tag{8}$$

It has been found that dynamic of considered chaotic system is described with third order differential equation. This equation has two summands which are shown in brackets. The first one is linear and the second one is nonlinear summands.

Later, linear part of (8) has been considered and defined at this following transfer function shown in (9).

$$W(s) = \frac{x_1(s)}{u(s)} = \frac{m_3}{s^3 + a_3 s^2 + a_2 s + a_1}. \tag{9}$$

Above-given transfer function has been used to simplify block-diagram of the considered system by using transformations of block-diagrams (Fig. 2).

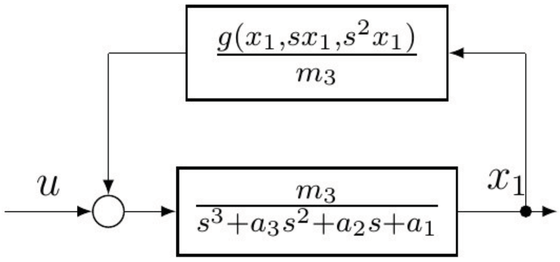

Fig. 2. 3-D chaotic system with linear feedforward and nonlinear feedback

Fig. 2 shows a chaotic system as serial 3-D chaotic system with the nonlinear feedback. Moreover, Block-diagram which are shown on Fig. 2 define the formulation of the following statement:

Statement 1. Dynamic of generalized third order chaotic system is defined by linear differential operator (9) in system's feedforward and nonlinear one (4) in it's feedback.

Since (3) has been assumed to have real negative eigenvalues, it can be concluded that linear feedforward is an asymptotically stable and chaotic oscillations caused by nonlinear feedback only.

Now, transfer function (9) can be transformed into parallel form. Since it has only three different eigenvalues, such transformation is trivial.

$$\frac{m_3}{s^3 + a_3 s^2 + a_2 s + a_1} = \frac{A_1}{s - \lambda_1} + \frac{A_2}{s - \lambda_2} + \frac{A_3}{s - \lambda_3}. \tag{10}$$

Equation (10) can be reduced to a common denominator and written down equations for defining unknown A_i coefficients.

$$\begin{aligned} &A_1 + A_2 + A_3 = 0; \\ &A_1(\lambda_2 + \lambda_3) + A_2(\lambda_1 + \lambda_3) + A_3(\lambda_1 + \lambda_2) = 0;. \\ &A_1\lambda_2\lambda_3 + A_2\lambda_1\lambda_3 + A_3\lambda_1\lambda_2 = m_3. \end{aligned} \tag{11}$$

Equation (11) has been solved for unknown A_i coefficients and put down this following expression.

$$\begin{aligned} A_1 &= -\frac{m_3}{\lambda_1^2 - \lambda_1\lambda_2 - \lambda_1\lambda_3 + \lambda_2\lambda_3}; \\ A_2 &= \frac{m_3}{-\lambda_1\lambda_2 - \lambda_1\lambda_3 - \lambda_2^2 + \lambda_2\lambda_3};. \\ A_3 &= -\frac{m_3}{\lambda_1\lambda_2 - \lambda_1\lambda_3 - \lambda_2\lambda_3 + \lambda_3^2}. \end{aligned} \tag{12}$$

Expression (10) makes it possible to transform block-diagram Fig. 2 as it is shown in Fig. 3.

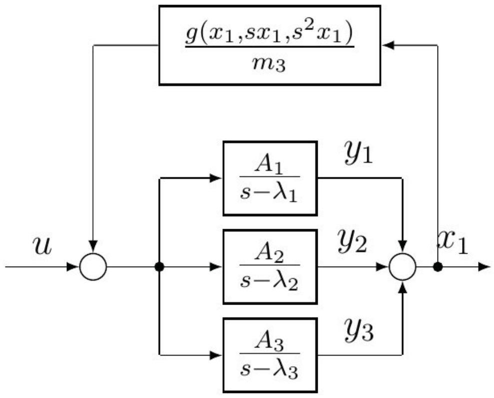

Fig. 3. 3-D chaotic system with linear parallel feedforward

Block-diagram in Fig. 3 and (10) emerges a possibility to write down new state space equations.

$$\begin{aligned} sy_1 &= \lambda_1 y_1 + k_1 g\left(x_1, sx_1, s^2 x_1\right) + A_1 u; \\ sy_2 &= \lambda_2 y_2 + k_2 g\left(x_1, sx_1, s^2 x_1\right) + A_2 u; \\ sy_3 &= \lambda_3 y_3 + k_3 g\left(x_1, sx_1, s^2 x_1\right) + A_3 u; \\ x_1 &= y_1 + y_2 + y_3, \end{aligned} \tag{13}$$

where

$$k_i = A_i / m_3 . \tag{14}$$

Dynamic system (13) has three parallel channels with inner linear feedbacks and outer nonlinear ones. It is clearly understood that outer feedback depends on the first and second derivatives of output variable x_1. Calculation of these derivatives is quite nontrivial problem.

B. Parallel observer for definition of state variables' derivatives

partial fraction decomposition of transfer function (9) has been offered not only for getting parallel model of dynamical objects, but also for calculation of derivatives from output variable as well.

Later, equation (9) differentiate between its left and right hand expressions. This operation in the operator form can be performed by multiplying the function on derivative operator.

$$W_1(s) = \frac{sx_1(s)}{u(s)} = \frac{m_3 s}{s^3 + a_3 s^2 + a_2 s + a_1}. \tag{15}$$

Transfer function (15) can be converted into parallel form by using above-described approach.

$$\frac{m_3 s}{s^3 + a_3 s^2 + a_2 s + a_1} = \frac{B_1}{s-\lambda_1} + \frac{B_2}{s-\lambda_2} + \frac{B_3}{s-\lambda_3}. \tag{16}$$

Coefficients B_i were defined after reducing right-hand expression (16) to a common denominator and write down equations for defining unknown B_i coefficients.

$$\begin{aligned} &B_1 + B_2 + B_3 = 0; \\ &B_1(\lambda_2 + \lambda_3) + B_2(\lambda_1 + \lambda_3) + B_3(\lambda_1 + \lambda_2) = m_3; \\ &B_1\lambda_2\lambda_3 + B_2\lambda_1\lambda_3 + B_3\lambda_1\lambda_2 = 0. \end{aligned} \tag{17}$$

Solution of (17) has put down the following expressions.

$$\begin{aligned} B_1 &= -\frac{\lambda_1 m_3}{\lambda_1^2 - \lambda_1\lambda_2 - \lambda_1\lambda_3 + \lambda_2\lambda_3}; \\ B_2 &= \frac{\lambda_2 m_3}{-\lambda_1\lambda_2 - \lambda_1\lambda_3 - \lambda_2^2 + \lambda_2\lambda_3}; \\ B_3 &= -\frac{\lambda_3 m_3}{\lambda_1\lambda_2 - \lambda_1\lambda_3 - \lambda_2\lambda_3 + \lambda_3^2}. \end{aligned} \tag{18}$$

The second derivative of output variable can be defined in a similar way by multiplying (15) on the derivative operator's image.

$$W_2(s) = sW_1(s) = \frac{s^2 x_1(s)}{u(s)} = \frac{m_3 s^2}{s^3 + a_3 s^2 + a_2 s + a_1} \tag{19}$$

and replacing transfer function (19) with (20).

$$\frac{m_3 s^2}{s^3 + a_3 s^2 + a_2 s + a_1} = \frac{C_1}{s-\lambda_1} + \frac{C_2}{s-\lambda_2} + \frac{C_3}{s-\lambda_3}. \tag{20}$$

Unknown coefficients C_i can be defined by solving (21).

$$\begin{aligned} &C_1 + C_2 + C_3 = m_3; \\ &C_1(\lambda_2 + \lambda_3) + C_2(\lambda_1 + \lambda_3) + C_3(\lambda_1 + \lambda_2) = 0; \\ &C_1\lambda_2\lambda_3 + C_2\lambda_1\lambda_3 + C_3\lambda_1\lambda_2 = 0 \end{aligned} \tag{21}$$

and writing down the solution as (22).

$$\begin{aligned} C_1 &= -\frac{\lambda_1^2 m_3}{\lambda_1^2 - \lambda_1\lambda_2 - \lambda_1\lambda_3 + \lambda_2\lambda_3}; \\ C_2 &= \frac{\lambda_2^2 m_3}{-\lambda_1\lambda_2 - \lambda_1\lambda_3 - \lambda_2^2 + \lambda_2\lambda_3}; \\ C_2 &= -\frac{\lambda_3^2 m_3}{\lambda_1\lambda_2 - \lambda_1\lambda_3 - \lambda_2\lambda_3 + \lambda_3^2}. \end{aligned} \tag{22}$$

Analysis of the above-given formulas helps to make the following statement.

Statement 2. When the model of the considered system is given in parallel form, the determination of derivatives from output state variable can be performed by using specially defined coefficients only.

Comparison of (12), (18), and (22) make it possible to generalize these coefficients and write down following recursive formulas.

$$K_{ij} = \lambda_i K_{i(j-1)} = \lambda_i^j A_i , \tag{23}$$

where i is a parallel channel number, j is a derivative's order.

Expression (23) can be rewritten (13) as follows.

$$\begin{aligned} sy_1 &= \lambda_1 y_1 + k_1 g\left(x_1, sx_1, s^2 x_1\right) + A_1 u; \\ sy_2 &= \lambda_2 y_2 + k_2 g\left(x_1, sx_1, s^2 x_1\right) + A_2 u; \\ sy_3 &= \lambda_3 y_3 + k_3 g\left(x_1, sx_1, s^2 x_1\right) + A_3 u; \\ x_1 &= y_1 + y_2 + y_3, sx_1 = \lambda_1 y_1 + \lambda_2 y_2 + \lambda_3 y_3, \\ s^2 x_1 &= \lambda_1^2 y_1 + \lambda_2^2 y_2 + \lambda_3^2 y_3. \end{aligned} \quad (24)$$

It is necessary to assume that the last three equations of (24) describe observer dynamic.

Fig.4 shows a block-diagram of chaotic system which is described with equations (24).

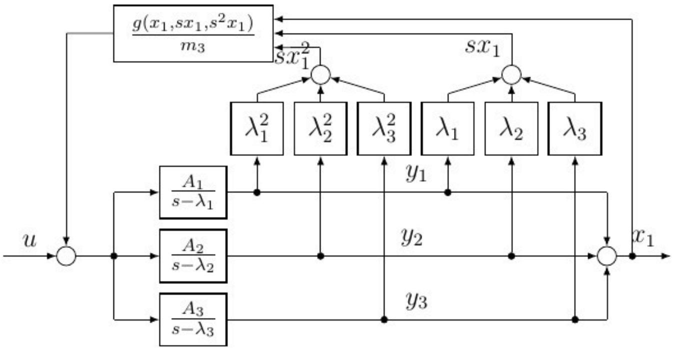

Fig. 4. Block-diagram of 3-D chaotic system with the parallel observer

Analysis of (24) makes it possible to conclude that the proposed approach constructed both mathematical model of considered system and observer for defining unknown state variables. Moreover, this following statement has been made.

Statement 3. Transformation of mathematical model into parallel form, which is performed by using partial fraction decomposition, defines that parallel mathematical model consists of three parts: linear feedforward, linear observer and nonlinear feedback.

This structure makes it possible to construct novel chaotic systems by studying feedback's properties.

III. Modeling and Simulation of 3-D Jerk Chaotic System with Three Quadratic Nonlinearities

The 3-D jerk controllable chaotic system has been now considered with quadratic nonlinearities.

$$\begin{aligned} &\dot{x}_1 = x_2; \dot{x}_2 = x_3; \\ &\dot{x}_3 = ax_1 - bx_2 - x_3 + cx_1x_2 - p\left(x_1^2 + x_2^2\right) + u, \end{aligned} \quad (25)$$

where

$$a = 7.5; b = 4; c = 0.03; p = 0.9. \quad (26)$$

A. Chaotic system's parallel modeling

It was assumed that desired eigenvalues were expressed as (27).

$$\lambda_1 = -1; \lambda_2 = -2; \lambda_3 = -3. \quad (27)$$

Usage of (5) defined the coefficients of the characteristic polynomial.

$$\begin{aligned} &a_1 = -\lambda_1\lambda_2\lambda_3 = 6; a_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = 11; \\ &a_3 = -\lambda_1 - \lambda_2 - \lambda_3 = 6 \end{aligned} \quad (28)$$

and rewrite (9) in such a way.

$$W(s) = \frac{x_1(s)}{u(s)} = \frac{1}{s^3 + 6_3 s^2 + 11_2 s + 6}. \quad (29)$$

expressions (12), (18) and (22) have been used for defining model's and observer parameters.

$$\begin{aligned} &A_1 = 0.5; A_2 = -1; A_3 = 0.5; \\ &B_1 = -0.5; B_2 = 2; B_3 = -1.5;. \\ &C_1 = 0.5; C_2 = -4; C_3 = 4.5. \end{aligned} \quad (30)$$

parameters (26) and (28) have been used to write down nonlinear function $g(x_1, x_2, x_3)$

$$\begin{aligned} &g(x_1, x_2, x_3) = 13.5x_1 + 7x_2 + 5x_3 + \\ &+ 0.03x_1x_2 - 0.9\left(x_1^2 + x_2^2\right) \end{aligned} \quad (31)$$

Parameter (29) can be used as feedforward and (31) as feedback while considered chaotic system is being modeled and simulated.

This way for studying the chaotic system is very convenient, but it requires using an observer for defining high derivatives. We can avoid observer's using by substituting into (31) their values from (24). This substitution rewrite parallel model (24) as follows.

$$\begin{aligned} &sy_1 = -0.915y_1^2 + (4.75 - 2.745y_2 - 3.66y_3)y_1 - \\ &- 2.28y_2^2 + (9.75 - 6.375y_3)y_2 + 18.75y_3 - 4.545y_3^2 + 0.5u; \\ &sy_2 = 1.83y_1^2 + (-11.5 + 5.49y_2 + 7.32y_3)y_1 + \\ &+ 4.56y_2^2 + (-21.5 + 12.75y_3)y_2 - 37.5y_3 + 9.09y_3^2 - u; \\ &sy_3 = -0.915y_1^2 + (5.75 - 2.745y_2 - 3.66y_3)y_1 - \\ &- 2.28y_2^2 + (9.75 - 6.375y_3)y_2 + 15.75y_3 - 4.545y_3^2 + 0.5u, \\ &x_1 = y_1 + y_2 + y_3. \end{aligned} \quad (32)$$

This model has three parallel channels but it is more complex than previous one and it has nonlinearities in every equation. Analysis of (32) shows that it has similar nonlinearities in the first and third equations which are differed only by linear summands near y_1 and y_3.

B. Chaotic system's dynamic studying

Dynamics of the considered system are now observed which are given in two ways: by (29) and (31), and by (32).

At first, it should be mentioned that all of three considered models which are described by (29) and (31), by (32), and by

(25) give us equal results with the high precision. These results are shown in Fig. 5 and errors Δx_i are multiplied on 1000.

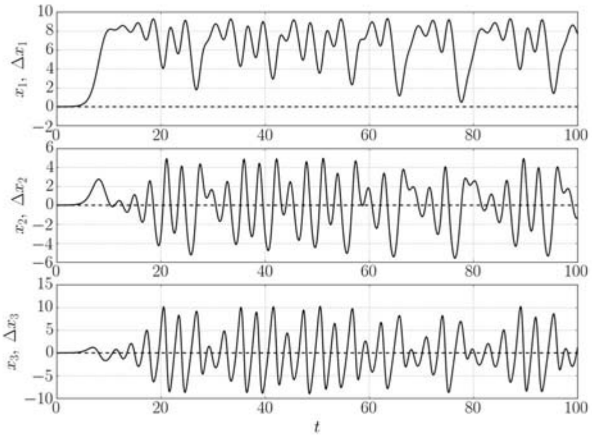

Fig. 5. Chaotic system's dynamic

Contrary to source model (25) models with and without observer produce some virtual state variables y_i (Fig. 6).

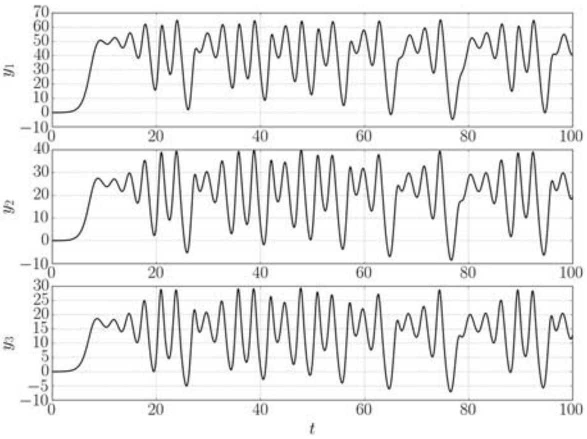

Fig. 6. Parallel system's dynamic in virtual state variables

It has been shown that above-mentioned virtual coordinates define the novel attractors (Fig. 7) which are differed from well-known one (Fig. 8).

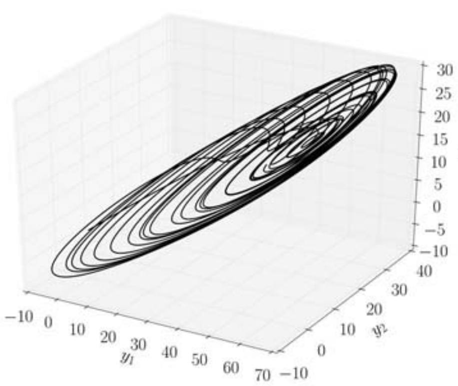

Fig. 7. Novel chaotic attractor of considered system

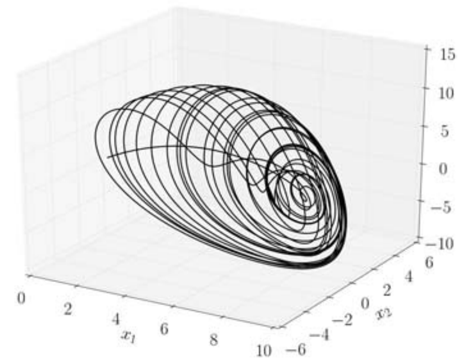

Fig. 8. Well-known chaotic attractor of considered system

Analysis of simulation result shows that chaotic system has unpredictable oscillations in each of state spaces which are used for describing of this system.

IV. CONCLUSIONS

Transformation of nonlinear system with chaotic dynamic into parallel form has several benefits. At first, it was performed in analytical way with high precision. This fact provides a similar system dynamic while different implementations are being used. Other benefit is the possibility to define feedforward's dynamic by assuming its desired eigenvalues. This fact makes its possible to produce chaotic oscillations in linear stable system by using nonlinear feedback. It also can be found that it is possible to get the desired dynamic by defining feedforward's one and performing compensation of nonlinear feedback. Finally, it is possible to generate various attractors which can be used for different technical and scientific applications.

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