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Editors

Uffe Thomas Jankvist, Danish School of Education, Aarhus University, Denmark.

Marja van den Heuvel-Panhuizen, Freudenthal Group & Freudenthal Institute, Utrecht University, Netherlands; Nord University, Norway.

Michiel Veldhuis, Freudenthal Group, Utrecht University, Netherlands; iPabo University of Applied Sciences, Amsterdam, Netherlands.

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Students' abilities on the relationship between beliefs and practices

Safrudiannur[1,2] and Benjamin Rott[2]

[1]Mulawarman University, Indonesia; safrudiannur.safrudiannur@uni-koeln.de

[2]University of Cologne, Germany; benjamin.rott@uni-koeln.de

Most studies that deal with possible influences of the social context related to students' abilities on the connection between beliefs and practices involve only a small number of participants (e.g., 1 – 3 teachers). Thus, we have designed a new quantitative questionnaire (named TBTP) which is intended to examine these influences for a larger number of teachers (here: N = 43). Then, for a further qualitative investigation, we interviewed and observed lessons of three of those teachers. The results show that the 43 teachers distinguish their style of teaching in different contexts of students' abilities. The correlation between beliefs about mathematics and the styles of teaching can be seen in the context of low ability students rather than in the context of high ability students. In the further investigation, this correlation is supported by the three teachers' descriptions of their general teaching of mathematics and their real practices in classes containing many low ability students.

Keywords: Teachers' beliefs, teaching and learning of mathematics, students' abilities.

Introduction

Some researchers have shown that there is a connection between teachers' beliefs and practices (see Philipp, 2007). Other researchers, however, have reported that there may be inconsistencies between teachers' beliefs and their practices (Raymond, 1997; see Philipp, 2007). Buehl and Beck (2015) argue that such inconsistencies should not be a reason to discount the power of beliefs, but that other factors like social contexts support or hinder the connection between beliefs and practice. Several studies (e.g., Raymond, 1997; Cross Francis, 2015) have revealed that social contexts in a classroom might cause teachers to act in a way that differs from the way that fits their beliefs. However, most of the studies revealing the influence of social contexts are case studies involving only a small number of teachers such as one, two, or three teachers (see Raymond, 1997; Cross Francis, 2015).

Studying teachers' beliefs and practices by doing intensive case studies involving interviews and observations may provide convincing data (Philipp, 2007), but they are too expensive for large samples of teachers. The use of self-report instruments is cost-effective, but their accuracy and validity are criticized (e.g., Di Martino & Sabena, 2010). Most self-report instruments measuring beliefs use closed questions, mainly Likert scale items. However, Di Martino and Sabena (2010) question the use of the Likert scale since it amplifies problems related to social desirability. The social desirability problem may arise when items being rated are viewed as inherently positive by respondents (McCarthy & Shrum, 1997). Thus, teachers' responses to rating or Likert scale items may reflect what is socially accepted and should be done rather than what actually is done (Fang, 1996). Furthermore, Likert scale items provide less or no contexts (Philipp, 2007), whereas, as we pointed out before, social contexts at school may affect teachers' practices in a classroom.

We offer an approach to minimize the social desirability problem and consider the social contexts at school in order to increase the accuracy of the prediction of teachers' beliefs and practices. We have developed a questionnaire for studying **t**eachers' **b**eliefs on **t**heir **p**ractice (named TBTP). We use rank-then-rate (a combination of ranking and rating) items. McCarthy and Shrum (1997) show that using rank-then-rate items may reduce respondents' tendency to give high ratings towards items viewed inherently positive socially. Moreover, we consider students' abilities as the social context in a classroom since several researchers (e.g., Raymond, 1997; Beswick, 2018) have shown that students' abilities may influence teachers' beliefs and practices.

We conduct this pilot study to evaluate whether the TBTP gives us insight into the relationship between beliefs and practices in different contexts of students' abilities. Ribeiro et al. (2019) suggest that considering beliefs and practices in diverse contexts may contribute for a better understanding on the beliefs and practices in order to improve the quality of teacher education.

Theoretical framework

Beliefs of the nature of mathematics and its association with teaching and learning of math

Philipp (2007, p. 259) defines "beliefs as psychologically held understandings, premises, or propositions about the world that are thought to be true". Teachers may hold various beliefs about mathematics since they may see mathematics with different views. Ernest (1989) summarizes three views about the nature of mathematics: *the instrumentalist view* (mathematics as an accumulation of facts and rules to be used in the pursuance of some external end), *the Platonist view* (mathematics as a static but unified body of knowledge), and *the problem-solving view* (mathematics as a dynamic process which is continually expanding field of human creation and invention). Nonetheless, Ernest (1989) argues that teachers may merge elements from more than one of the three views.

Ernest (1989) further argues that the three views about the nature of mathematics can be associated with the models of teaching mathematics: (1) the instrumentalist view is linked with the role as an instructor who demonstrates math skills correctly; (2) the Platonist view is linked with the role as an explainer who describes the relation of concepts; and (3) the problem-solving view is linked with the role as a facilitator who likes doing problem solving or posing activities.

Students' abilities as a social context

Empirically, Ernest's association between beliefs about mathematics and teaching of mathematics has not been demonstrated well. Ernest (1989) argues that social contexts such as students' behaviors may cause inconsistencies between teachers' beliefs and practices. Raymond (1997) has shown that students' abilities, attitudes, and behaviors have strong influences on teachers' practice. Moreover, Zohar, Degani, and Vaaknin (2001) found that most teachers believe that teaching with higher order thinking is only appropriate for high-achieving students, not for low-achieving students. Therefore, in this study, we consider students' abilities as the social context which may affect teachers' practices.

Research questions and method

As pointed out before, studies revealing the influence of the social context on teachers' practices involved only small numbers of teachers. In this study, we will examine the influence of the social context related to students' abilities for a larger number. The questions are: (1) How and why do teachers differentiate their style of teaching and learning of mathematics because of students' abilities? And (2) how do students' abilities influence the relationship between teachers' beliefs about mathematics and their styles of teaching? To answer the questions, we use a multi-method design by combining the TBTP questionnaire with interviews and classroom observations.

The TBTP. This questionnaire has ten rank-then-rate items grouped into three themes presented in the Appendix (see Safrudiannur & Rott, 2018, for the evaluation of the reliability and validity of the TBTP). In this paper, since we focus on teaching of mathematics, we only discuss teachers' responses to items of Themes 1 and 3. Each item consists of three statements. The first, second, and third statement are always associated with the instrumentalist, the Platonist, and the problem-solving views, respectively. To consider students' abilities as the social context, the items of Theme 1 are posed twice: for classes dominated by high ability (HA) and by low ability (LA) students (see Figure 1). Please note that we define the terms HA and LA by using the students' achievements (see Appendix), following Zohar et al. (2001) who use the terms achievement and ability interchangeably.

Item 1: When you teach the formula in a class dominated by **HA students**, what is important for you?							
RANK							
Please order the three statements below by giving a rank 1 (the most important), 2, or 3 (the least important).							
Statements							**Rank**
R1. You demonstrate how to use the formula correctly by giving some examples.							3
R2. You explain concepts related to how to get or to prove the formula.							2
R3. You let your students discover the formulas in their own ways.							1
RATE BASED ON YOUR RANK ABOVE							
Please rate the level of importance of each statement based on your rank above.							
Statements	**Rate**						
Statement R1	1	(2)	3	4	5	6	7
Statement R2	1	2	3	(4)	5	6	7
Statement R3	1	2	3	4	(5)	6	7

Item 2: When you teach the formula in a class dominated by **LA students**, what is important for you?							
RANK							
Please order the three statements.							
Statements							**Rank**
Statement R1							2
Statement R2							1
Statement R3							3
RATE BASED ON YOUR RANK ABOVE							
Please rate the level of importance of each statement.							
Statements	**Rate**						
Statement R1	1	2	3	4	(5)	6	7
Statement R2	1	2	3	4	5	6	(7)
Statement R3	1	(2)	3	4	5	6	7

Figure 1: An example how Fitria responded to item 1 and item 2 (identical statements) of the TBTP

To respond to an item, a respondent firstly orders the three statements of the item by assigning a rank 1, 2, or 3, and then rate each of them from 1 to 7 based on his ranks (see Figure 1). Thus, there will be two sets of data: ranking and rating data. For data analyses, we only use rating data, because ranking data is included. The purpose of the ranking part is only to make respondents discriminate three statements of each item before rating them and to minimize the impact of the social desirability.

We asked 43 Indonesian math teachers who accompanied their students in a math competition to respond to the TBTP (their background in detail, sex: 33 females, 7 males, and 3 did not state; school: 14 from primary, 16 from lower secondary, 10 from upper secondary, and 3 did not state; teaching years: 7 for less than two years, 10 in between two and five years, 9 in between five and ten years, 16 for more than ten years, and 1 did not state).

Interviews and Observations. Three (all from lower secondary) of the 43 teachers volunteered for a further investigation: Elisa (female), Fitria (female), and Dony (male). Elisa and Dony were math teachers for two to five years, and Fitria for more than ten years. We chose them since they could represent lessons in HA and LA classes. The interviews aimed at finding information about their beliefs about mathematics and general teaching of mathematics. We identified key words/ statements reflecting their beliefs. We also videotaped one lesson from each teacher. We used the coding system from the TIMSS Video Study 1999 to interpret the lessons. We categorized the interaction (*Public*, *Mix*, *Private*) and the content activity (*Non-problem* or *Problem Segment*) of the lesson (we explain each term directly in the description of each lesson in the result section to save space). Both authors of this paper and one expert from Indonesia made the interpretation of interviews and lessons. The lowest percentage agreement is 70 % (we did discussions [consensual validation] to solve disagreements).

Results

Responses to the TBTP questionnaire

Tables 1 and 2 summarizes the responses of all 43 teachers to the TBTP. The results of paired t-tests show that teachers significantly differ their reported styles of teaching between HA and LA classes. Table 1 shows that teachers gave higher rates to the statements associated with the instrumentalist view for LA classes than to those for HA classes. In contrast, the statements associated with the problem-solving view for LA classes have lower rates than those for HA classes.

No	Statements of Theme 1 (views associated with the statements)	Teachers (n=43)		t-values (df=42, two tailed)	Elisa		Fitria		Doni	
		HA class Mean (sd)	LA class Mean (sd)		HA	LA	HA	LA	HA	LA
1 and 2	R1 (Instrumentalist view)	4.09 (1.66)	5.65 (1.13)	-6.19[a]	5	7	2	5	2	6
	R2 (Platonist view)	5.02 (1.39)	5.44 (1.42)	-1.46	6	5	4	7	4	4
	R3 (Problem-solving view)	5.33 (1.55)	3.47 (1.24)	6.35[a]	4	4	5	2	6	2
3 and 4	S1 (Instrumentalist view)	4.07 (1.49)	5.35 (1.15)	-4.99[a]	3	5	2	5	5	6
	S2 (Platonist view)	5.86 (1.19)	5.47 (1.32)	1.57	7	7	5	7	6	3
	S3 (Problem-solving view)	5.44 (1.05)	3.79 (1.81)	5.52[a]	6	4	6	4	4	1

[a] significant for $p < 0.004$ (The adjustment of alpha = 0.05 by Bonferroni's correction for 12 multiple t-test)

Table 1: Teachers' responses to items of Theme 1 and the results of the paired sample t-test

There are no differences between rates related to the statements regarding the Platonist view. This indicates that explanation and understanding for both HA and LA classes are essential to the teachers.

No	Statements of Theme 3 (views associated with the statements)	Teachers (n=43)	Correlations with statements of Theme 1 in the same view		Elisa	Fitri	Doni
		Mean (sd)	HA class	LA class			
9	P1 (Instrumentalist view)	5.37 (1.43)	R1 (0.29), S1 (0.39[b])	R1 (0.42[b]), S1 (0.34[a])	6	6	5
	P2 (Platonist view)	4.95 (1.19)	R2 (0.30[a]), S2 (0.28)	R2 (0.18), S2 (0.33[a])	5	5	3
	P3 (Problem-solving view)	4.70 (1.39)	R3 (0.28), S3 (0.26)	R3 (0.25), S3 (0.20)	4	4	2
10	Q1 (Instrumentalist view)	5.09 (1.54)	R1 (0.07), S1 (0.29)	R1 (0.44[b]), S1 (0.53[b])	5	6	6
	Q2 (Platonist view)	5.14 (1.49)	R2 (0.33[a]), S2 (0.36[a])	R2 (0.32[a]), S2 (0.39[b])	3	4	3
	Q3 (Problem solving view)	4.42 (1.26)	R3 (0.09), S3 (0.07)	R3 (0.53[b]), S3 (0.54[b])	4	3	5

[a] significant for $p < 0.05$; [b] significant for $p < 0.01$

Table 2: Teachers' responses to items of Theme 3 and the results of the Pearson correlation

Table 2 summarizes the responses of all 43 teachers' to the two items of Theme 3. Most of the teachers gave higher rates to statements associated with the instrumentalist view (P1 and Q1). Interestingly, the correlation analyses indicate that the rates to the statements of Theme 3 (about the nature of mathematics) significantly correlate to the rates to the statements of Theme 1 (about teaching and learning of mathematics) for LA classes, particularly in the instrumentalist view.

Interviews

All three teachers' reports about their beliefs of the nature of mathematics seem to be in line with their responses to the items of Theme 3 of the TBTP presented in Table 2 (all gave higher rates to both statements P1 and Q1 associated with the instrumentalist view than to the other statements).

> Elisa: I often meet people who love mathematics so much. I think, I am not like that. I am amazed by people who can (…) such as (…) can understand fast, and fast in solving problems or anything. I am not, honestly, I understand slowly. So, I think, math is a collection of facts, formulas, which make me confused.

Fitria also expressed that mathematics consists of a lot of formulas and rules which are useful for solving problems and their truth is absolute. Doni also expressed that the truth of mathematics is absolute since its contents result from absolute agreements which could not be changed. He stressed that mathematics is useful for humans for solving problems and that it can be applied universally. Additionally, all three interviewees emphasized the importance of memorizing math formulas for their students. These expressions seem to indicate that they dominantly hold the instrumentalist view since they view mathematics as a toolbox consisting of utilitarian facts and rules to be used by the skillfully trained artisan in the pursuance of some external end (Ernest, 1989).

All interviewees described that they usually taught mathematics by demonstrating formulas and giving examples (in line with their responses to items of Theme 1 for LA classes, see Table 1). They spent lots of time providing many examples and often repeated their explanation to ensure that students understood mathematics. They assessed that many of their students were LA students. They argued that if their classes were dominated by HA students, they would be able to apply their ideal teaching. They believed that only HA students could discover mathematical formulas by themselves.

> Doni: Ideally, teaching in HA class. In my class, the abilities are heterogenous. I tend to… just did ordinary teaching. Giving them examples (…) cases (…) basic examples.

Unlike Elisa and Fitria who rarely met HA students, Doni reported that he had a club whose members were HA students from different classes. These students were trained to follow mathematics competitions. In the club, he gave his HA students difficult math tasks taken from math competitions.

Observed Lessons

Elisa taught her students solving a geometry task in the videotaped lesson in her LA class. Before handing out the task, a non-problem (NP) segment (*a segment containing math information but no tasks/problems*) took place. The interaction during this NP segment was coded as Public (*a public dialogue conducted by the teacher and other students must listen to it*) since Elisa led students to recall some math formulas and concepts. Public interactions also dominated the interaction during the problem segment (PS, *a segment containing math tasks/problems*) since Elisa often asked her students questions to make them recall some math formulas which were useful to solve the task. If students did not recall formulas, she reminded them of the formulas. Moreover, Elisa demonstrated, slowly explained, and guided students on how to use the formulas in the observed lesson.

There was no NP segment in Fitria's lesson in her LA class. Fitria taught about the application of mathematical operation sets in the real world by giving four tasks. The interaction during the PS segment of the first two tasks was only Public since Fitria demonstrated and explained the way to solve the two tasks. The interaction during the PS segment of the last two tasks was Private (*students individually worked on the two tasks*). However, during the Private interaction, Fitria reminded the students that they needed to refer to the way of solving the first two tasks, to be able to solve the last two. Moreover, when students got stuck, Fitria gave them mathematical clues to help them.

There was also no NP segment in Doni's lesson in his club (HA class). At the beginning of PS, Doni gave his HA students a sheet with 25 tasks. After that, he let students work individually (only Private interaction) without providing any clues to help them. After a break, Doni showed his students how to solve some of the tasks from the sheet, and the students checked their answers (Public interaction). During the show, Doni emphasized some math formulas that were important for students to memorize since the formulas were often used for solving tasks in mathematics competitions.

Discussion and Conclusion

In this study, we examine the influence of social contexts related to students' abilities in a classroom on a large number of teachers (43 teachers). The results indicate that students' abilities have an impact on the link between teachers' beliefs about mathematics and the style of teaching mathematics.

Firstly, we found that the 43 teachers report significantly different styles of teaching mathematics between HA and LA classes (Table 1), indicating that students' abilities influence teachers' beliefs and practices (Beswick, 1998; Raymond, 1997). The responses to items of Theme 1 for LA classes show that most of the participants gave high rates to both statements (R1 and S1) associated with the instrumentalist view. Interestingly, they gave low rates to both statements (R3 and S3) associated with the problem-solving view. In contrast, they rated R3 and S3 for HA classes higher than they did for LA classes. The three teachers in the further investigation believed that only HA students could discover math formulas by themselves, whereas LA students would understand mathematics slowly and should memorize mathematical formulas. Without memorizing, LA students could not solve math tasks or problems in their opinions.

However, Table 1 also indicates that teachers do not distinguish their teaching styles related to the Platonist view. The average rates of R2 and S2 are high for both HA and LA classes. This indicates that explanation is an essential part of their teaching, and generally, teachers want their students, both HA and LA students, to understand what they teach (Van de Walle et al., 2013).

Secondly, we found that 43 teachers' rates to statements about the nature of mathematics (Theme 3) correlate with their rates to statements about teaching and learning of mathematics (Theme 1) for LA classes (see Table 2), particularly on the instrumentalist view. Our further analyses with the three teachers seem to support this correlation. Table 2 indicates that those three teachers' rates about teaching and learning of mathematics for LA classes reflect what they believed about mathematics. For example, Elisa and Fitria seemed to hold the instrumentalist view dominantly. Their high rates to R1 and S1 of the TBTP for LA classes (see Table 1) seem to reflect their instrumentalist view of mathematics. Further, their descriptions of their general teaching, as well as their actual practices, also indicate the influences of the instrumentalist views since they played a role as an instructor who demonstrates skills correctly (Ernest, 1989).

In contrast, the three teachers' responses to Theme 1 for HA classes (see Table 1) which seem to be in line with their ideal teaching may not reflect their beliefs about mathematics. Although they seemed to hold the instrumentalist view, they gave low rates to R1 or S1 associated with the instrumentalist view but high rates to R3 or S3 associated with the problem-solving view. However, we still can see the impact of the instrumentalist view from the responses; e.g., Doni's high rates to S1 indicate the significance of memorizing formulas for HA students. In the observed lesson, he emphasized that his HA students should be able to memorize some formulas. This fact confirms the indication.

There are some limitations in this study. First, all three teachers in the qualitative investigation dominantly hold the instrumentalist view. Second, the characteristics of tasks used in the TBTP may not cover the complexity of teaching mathematics. For example, the task of item 1 (or item 2) only provides three statements about lesson learning a formula in the geometry topic (see Appendix), whereas, the way how teachers give their lessons in their own classes may be more complex than those three statements. Therefore, since this study is our pilot study and we plan to continue this study with larger number of teachers, we are going to add rank-then-rate items from other math topics and open questions in the TBTP. Furthermore, in the qualitative investigation, we will involve teachers from other views of mathematics (besides the instrumentalist view) in order to have better insight into the influence of students' abilities on the relationship between beliefs and practices.

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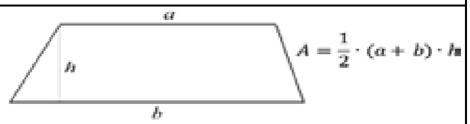
Appendix

General note: As a mathematics teacher, you have experience with high and low ability students in mathematics. Consider these definitions:

A high ability (HA) student is a student who generally shows a good understanding in your lessons and regularly has high scores in your tests.

A low ability (LA) student is a student who generally does not show a good understanding in your lessons and often has low scores in your tests.

Theme 1 (teaching and learning of mathematics): You are going to teach a lesson learning the formula to calculate the area of a trapezoid.
Please imagine this situation to answer the items 1 to 4.



Items 1 and 2*: When you have the lesson in an HA/LA class, what do you think that is important for you?
R1. You demonstrate how to use the formula correctly by giving some examples (*the instrumentalist view*).
R2. You explain concepts related to how to get or to prove the formula (*the Platonist view*).
R3. You let your students discover the formula in their own ways (*the problem-solving view*).
Items 3 and 4*: When you have the lesson in an HA/LA class, what do you think that is important for students?
S1. They memorize and use the formula correctly (*the instrumentalist view*).
S2. They understand the concepts underlying the formula from your explanation (*the Platonist view*).
S3. They can draw logical conclusions to deduce the formula (*the problem-solving view*).

Theme 2 (teaching and learning of problem solving): items 5-8, not discussed in this study.

Theme 3 (the nature of mathematics): Mathematics contents taught at school can be divided into several sub-domains such as numbers, algebra, geometry, measurements, statistics, and probability. The classifications of mathematics contents, in general, are more complicated, for example, classical algebra, linear algebra, number theory, differential geometry, calculus, statistics, probability theory, etc.
Item 9: In general, what do you think of the contents of mathematics?
P1. Mathematics is an accumulation of facts and skills, which are useful for human life. (*the instrumentalist view*)
P2. The contents are interrelated and connected within an organizational structure (*the Platonist view*).
P3. Mathematics is a dynamic process of human activities. The contents of mathematics expand and change to accommodate new developments (*the problem-solving view*).
Item 10: What do you think of the truth of the contents of mathematics?
Q1. The truth is absolute. The contents are free of ambiguity and conflicting interpretations (*the instrumentalist view*).
Q2. Mathematical ideas are pre-existing; humans just discover the contents of mathematics. Thus, the truth-value of mathematics is objective, not determined by humans (Q2, *the Platonist view*).
Q3. The contents are created by human and therefore, their truth-value is also established by humans (Q3, *prob. sol.*).

* *Items 1 = 2 and 3 =4, but with different classes (1 and 3 for HA, 2 and 4 for LA classes). See Figure 1.*