Bi-response Poisson Regression Model based on Local Linear for Modeling Effect of Early Marriage on The Child-Girl to Maternal and Infant Mortality in East Java

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Abstract. Early married on the child-girl is defined as a marriage of a girl before the age of 18. Early married has direct effects to girl education, health, and psychological so that increase risk of the maternal mortality and child mortality cases. The maternal mortality and child mortality cases can also be an indicator of the welfare level of a region. If the maternal mortality and the infant maternal cases are the high, then the quality of health services in the area is bad and conversely. In this paper, we discuss about effect of early marriage on the child-girl to maternal and infant mortality in East Java Indonesia using bi-response Poisson regression model based on local linear. The best model is determined based on Cross Validation (CV). The bi-response Poisson regression model with the best local linear estimator is obtained from optimal bandwidth is 0.67. The comparison of the deviance value between parametric and nonparametric regression model in this case showed the bi-response Poisson regression using nonparametric regression model approach based on local linear estimator is better than the bi-response Poisson regression using parametric regression approach. The analysis showed the early marriage on the child-girl had the highest influence on the number of maternal and infant mortality in East Java.

INTRODUCTION

States that in many developing countries, transition from adolescent to adulthood is abruptly cut short and the fundamental rights of female adolescents are compromised by early marriage, a practice that has serious consequences for their health and development. Early marriage is an appalling violation of human rights and robs girls of their education, health and long-term prospects early marriage, sometimes referred to as child marriage is defined as a marriage of a girl or boy before the age of 18 and refers to both formal marriages and informal unions in which children under the age of 18 live with a partner as if married (UNICEF [1]). According to Nour [2], a human rights violation, child marriage directly affects girls' education, health, psychological well-being, and the health of their offspring. Child marriage and early pregnancy affects women's general health, their productivity, job opportunities and prospects for escaping poverty. The notion of good reproductive health covers all aspects of the reproduction process including a satisfying and safe experience of sexual relations, capability to reproduce, and the freedom to decide if and when to bear a child (ICPD [3]). The right not to engage in sexual relations and the right to exercise control over reproduction may both be violated by early marriage. Setiawan [4] using parametric approach that is geographically weighted bivariate generalized Poisson regression shown that the early marriage on the child-girl influence to maternal mortality and child mortality in East Java Province Indonesia.

According to the Health Department of East Java (Dinas Kesehatan Jawa Timur) [5], the maternal mortality is the death of woman while pregnant or within 42 days of termination of pregnancy, irrespective of the duration and site of the pregnancy, from any cause related to or aggravated by the pregnancy or its management but not from accidental or incidental causes. The infant mortality is the death of an infant after child first birthday but child under five year of age. Indonesia's progress on maternal health, the fifth Millennium Development Goals (MDGs), has slowed in recent years. In East Java, the maternal mortality in 2015 is 89.6 per 100.000 live births and it has increased to 91.0 per

100.000 live births in 2016. Every three minutes, a child under the age of five years is dies and every hour, a woman dies from giving or of causes related to pregnancy.

This research was analyzed effect of early marriage on the child-girl to maternal and infant mortality in East Java province Indonesia using nonparametric approach that is bi-response Poisson regression based on local linear. Poisson regression model has been widely used for public health in recent years to analysis the cause of a disease (Darnah *et. al.*, [6]). For instance, Darnah [7] proposed model to know the factors influence of filariasis in East Java Indonesia, Asrul & Naing [8] used Poisson regression for modeling of AIDS disease in Kelantan Malaysia and then comparing it with the Binomial Negative regression. Darnah *et. al.* [6] has modeling maternal mortality and infant mortality cases in East Kalimantan Indonesia using Poisson regression approach based on local linear estimator.

Local linear estimator is widely used to estimating regression function because it is simple, easy to apply, appropriate for fixed or random design, and fits well to boundaries. Local linear estimator is obtained by locally fitting degree polynomial is one to data by using weighted least square. The local linear estimator depends on the bandwidth parameter that control smoothness of the fit [9].

MATERIAL AND METHODS

In this chapter, we discuss about the theoretical model for bi-response Poisson regression model based on local linear and the data used for known effect of early marriage on the child-girl to maternal mortality and child mortality in East Java Indonesia.

Bi-response Poisson Regression Models

Let the three random variables V_1, V_2, and U to follow three independent Poisson distributions with the positive parameters θ_1, θ_2, and γ respectively. According to Jung & Winkelmann [10], new random variables Y_1 and Y_2 can be constructed by $Y_1 = V_1 + U$ and $Y_2 = V_2 + U$ where Y_1 and Y_2 are Poisson random variables. The mean Y_1 and Y_2 are $E(Y_1) = \theta_1 + \gamma$ and $E(Y_2) = \theta_2 + \gamma$. The density probability function of Y_1 and Y_2 given by:

$$\mathrm{P}(\mathrm{Y}_1 = \mathrm{y}_1, \mathrm{Y}_2 = \mathrm{y}_2) = f(y_1, y_2) = \exp\left[-(\theta_1 + \theta_2 + \gamma)\right] \sum_{k=0}^{s} \frac{\gamma^k}{k!} \frac{\theta_1^{y_1 - k}}{(y_1 - k)!} \frac{\theta_2^{y_2 - k}}{(y_2 - k)!}; \tag{1}$$

where $\mathrm{y}_1, \mathrm{y}_2 = 0, 1, 2, \ldots$ and $\mathrm{s} = \min(\mathrm{y}_1, \mathrm{y}_2)$.

Following the standard approach in univariate Poisson regression we model the marginal expectation of Y_1 and Y_2, respectively, as a loglinear function of exogenous variables.

$$\theta_{ri} + \gamma = \exp(\underset{\sim}{x}_{ri}^T \underset{\sim}{\beta}_r); r = 1, 2 \text{ dan } i = 1, 2, \ldots, n \tag{2}$$

The log likelihood function for the observed random sample is given by:

$$\mathrm{L} = n\gamma - \sum_{i=1}^{n} \exp(\underset{\sim}{x}_{1i}^T \underset{\sim}{\beta}^{(1)}) - \sum_{i=1}^{n} \exp(\underset{\sim}{x}_{2i}^T \underset{\sim}{\beta}^{(2)}) + \sum_{i=1}^{n} \log B_i; \tag{3}$$

Where:

$$B_i = \sum_{k=0}^{s_i} \frac{\gamma^k}{k!} \frac{[\exp(\underset{\sim}{x}_{1i}^T \underset{\sim}{\beta}^{(1)}) - \gamma]^{y_{1i} - k}}{(y_{1i} - k)!} \frac{[\exp(\underset{\sim}{x}_{2i}^T \underset{\sim}{\beta}^{(2)}) - \gamma]^{y_{2i} - k}}{(y_{2i} - k)!}; \; s_i = \min(y_{ri}), r = 1, 2$$

The maximum likelihood estimates of the parameters can be obtained by solving equations:

$$\frac{\partial \log L}{\partial \gamma} = 0 \text{ and } \frac{\partial \log L}{\partial \underset{\sim}{\beta}^{(r)}} = 0; \mathrm{r} = 1, 2.$$

Bi-response Poisson Regression Models using Local Linear Estimator

Suppose we have pair observational data (x_i, y_{1i}, y_{2i}), $r = 1,2$ dan $i = 1, 2, ..., n$ which is distribute independently with x is vector of covariates and y_{ri} is the count bi-response that follows the Poisson distribution. The probability density function of y_{ri} given by equation (1). Generally, equation (2) can be written:

$$\theta_{ri} + \gamma = \exp[m_r(x_i)]; r = 1,2 \text{ dan } i = 1, 2, ..., n \tag{4}$$

The function $m_r(.)$ in equation (4) is a smooth function. Assume that the function $m_r(.)$ has a $(p+1)^{th}$ continuous derivative at the point x_0. We approximate the function $m_r(.)$ by Taylor expansion with order one or $p = 1$, for the data point x_i in around of x_0 with $x_i \in (x_0 - h, x_0 + h)$ and h is a bandwidth:

$m_r(x_i) \approx \beta_{0i}^{(r)}(x_0) + \beta_{1i}^{(r)}(x_0)(x_i - x_0); r = 1,2$ or $m_r(x_i) = \underset{\sim}{x}_i^T(x_0)\underset{\sim}{\beta}^{(r)}(x_0); r = 1,2.$

Where:

$\underset{\sim}{x}_i^T = \begin{bmatrix} 1 & (x_i - x_0) \end{bmatrix}$ and $\underset{\sim}{\beta}^{(r)} = \begin{bmatrix} \beta_{0i}^{(r)}(x_0) & \beta_{1i}^{(r)}(x_0) \end{bmatrix}^T$

Let $K_h(x_i - x_0)$ and h are Kernel weight and bandwidth, respectively, the local likelihood function can be obtained from Eq. (1):

$$\ell(\theta_{1i}, \theta_{2i}, \gamma, x_0) = \prod_{i=1}^{n} f(y_{1i}, y_{2i})^{K_h(x_i - x_0)} \tag{5}$$

The log local likelihood function is:

$$\begin{aligned} \mathrm{L}(\theta_{1i}, \theta_{2i}, \gamma, x_0) &= \ln \ell(\theta_{1i}, \theta_{2i}, \gamma, x_0) \\ &= \sum_{i=1}^{n} \left(K_h(x_i - x_0) \left\{ \left[-(\theta_{1i} + \theta_{2i} + \gamma) \right] + \ln \sum_{k=0}^{s} \frac{\gamma^k}{k!} \frac{\theta_{1i}^{y_{1i}-k}}{(y_{1i}-k)!} \frac{\theta_{2i}^{y_{2i}-k}}{(y_{2i}-k)!} \right\} \right) \end{aligned} \tag{6}$$

By assuming bi-variate Poisson distribution for response variable y_{ri}, these considerations yield the conditional local weighted log-likelihood in Equation (6), the maximum likelihood estimator for parameter can be found from a solution of maximum likelihood equation [11]:

$$\frac{\partial L}{\partial \gamma} = \sum_{i=1}^{n} K_h(x_i - x_0) \left\{ 1 - \sum_{k=0}^{s} \left[\left(\frac{\exp\left[\beta_{0i}^{(1)}(x_0) + \beta_{1i}^{(1)}(x_i - x_0)\right] - \gamma y_{1i}}{\exp\left[\beta_{0i}^{(1)}(x_0) + \beta_{1i}^{(1)}(x_i - x_0)\right] - \gamma^2} \right) \frac{k}{\gamma} + \left(\frac{k - y_{2i}}{\exp\left[\beta_{0i}^{(2)}(x_0) + \beta_{1i}^{(2)}(x_i - x_0)\right] - \gamma} \right) \right] \right\} \tag{7}$$

$$\frac{\partial L}{\partial \underset{\sim}{\beta}^{(1)}} = \sum_{i=1}^{n} K_h(x_i - x_0) \left\{ -\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(1)}\right] \underset{\sim}{x}_i + \sum_{k=0}^{s} \frac{(y_{1i} - k)\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(1)}\right] \underset{\sim}{x}_i}{\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(1)}\right] - \gamma} \right\} \tag{8}$$

$$\frac{\partial L}{\partial \underset{\sim}{\beta}^{(2)}} = \sum_{i=1}^{n} K_h(x_i - x_0) \left\{ -\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(2)}\right] \underset{\sim}{x}_i + \sum_{k=0}^{s} \frac{(y_{1i} - k)(y_{2i} - k)\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(2)}\right] \underset{\sim}{x}_i}{\exp\left[\underset{\sim}{x}_i^T \underset{\sim}{\beta}^{(2)}\right] - \gamma} \right\} \tag{9}$$

The estimates of $\underset{\sim}{\beta}^{(2)}$ and γ can be solved by using the Iteratively Weighted Least Square (ILWS) procedure. The equation (7) to equation (9) depends on the bandwidth h. The bandwidth h controls smoothness of the fit. If h is too small, the fit becomes too noisy and the variance increases. Conversely, if h is too large, the fit becomes over-smoothed and important feature may be distorted or lost completely.

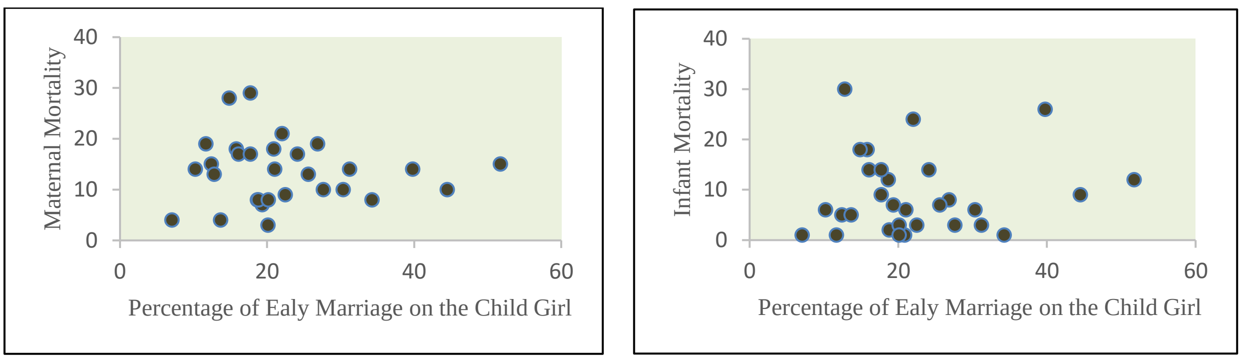
The Data

The data used in this research is secondary data, i.e., the maternal mortality and infant mortality cases in East Java 2017 as a response variable recorded by East Java Health Profile (Health Department of East Java / Dinas Kesehatan Jawa Timur [5]) and the predictor variable is the percentage of early marriage on the child-girl recorded by Statistical Bureau (BPS) [12] of East Java. East Java province is divided into 38 regencies. We use 30 regencies as in sample to modeling of maternal mortality and infant mortality, 8 regencies as out sample to prediction.

For analysis the data using bi-response Poisson regression based on local linear, we create R-code through the following steps. The first step is estimate parameter bi-response Poisson regression of local linear model by using weighted kernel locally maximum likelihood method. It can be done by iteratively reweighted least squares (IRLS). The second step is calculating Cross Validation (CV) to determine the best model. The CV criterion that has minimum value is the best model. The last step is interpretation of result based on the best model.

RESULTS AND DISCUSSION

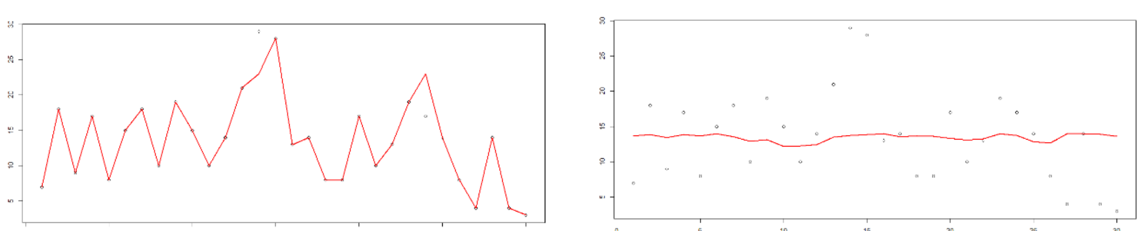
The scatter plot of total number of the maternal mortality and infant maternal mortality case by the percentage of early marriage on the child-girl is given in Fig. 1.



(a) (b)

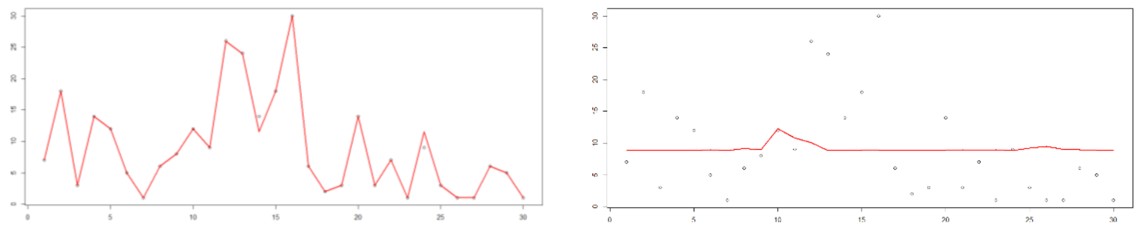
FIGURE 1. Plot of the Maternal Mortality Case (a) and Infant Mortality Case (b) by Percentage of Early Marriage on the Child Girl

Figure 1 show that the plot of the maternal mortality and child mortality case by percentage of early marriage on the child-girl in East Java not follow a particular pattern but tends to be irregular. Therefore, we apply the local linear estimator for bi-response Poisson regression to the data. The result local linear estimates of bi-response Poisson regression for maternal mortality and infant mortality with different bandwidth value is given in Fig. 2 and Fig. 3, respectively.



(a) (b)

FIGURE 2. Local Linear Estimates of Bi-response Poisson for maternal mortality case with $h = 0.01$ (a) & $h = 20$ (b).



(a) (b)

FIGURE 3 . Local Linear Estimates of Bi-response Poisson for Infant Mortality Case with h = 0.01 (a) & h = 20 (b).

According to Fig. 2 and Fig. 3, when the bandwidth value is small (h = 0.01), the regression curve is similar plots the maternal mortality and infant mortality data. Otherwise, when the bandwidth value is large (h = 20), the regression curve is over smooth. So that, we must to optimum bandwidth selection using CV criterion that has minimum value. Plot of CV versus bandwidth of both maternal and infant mortalities are showed in Figure 4.

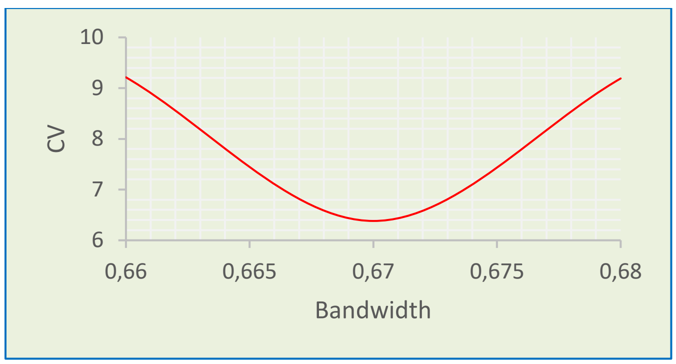
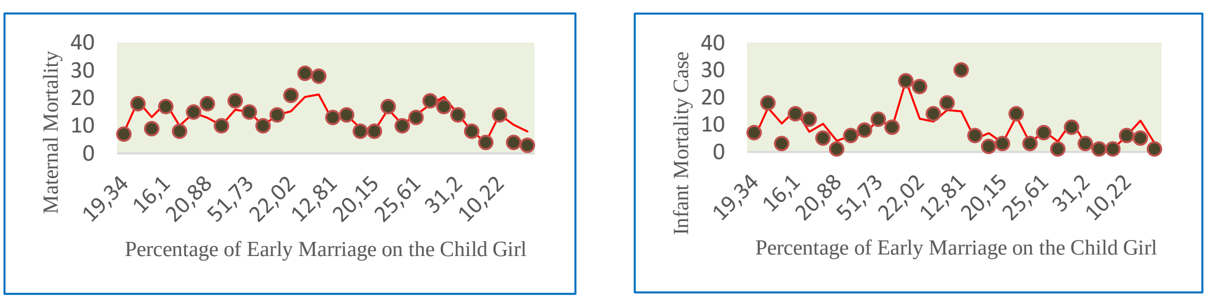


FIGURE 4. Plot of CV with bandwidth value.

According to Fig. 4**,** the optimum bandwidth is 0.67 and CV value is 6.381427. The optimum bandwidth used to get the best bi-response Poisson regression model. The best model for both responses is shown in Fig. 5.



(a) (b)

FIGURE 5. Estimation of maternal mortality (a) and infant mortality (b).

In bi-response Poisson regression modeling with a linear local approach, the estimated regression curves obtained are local so the results are different for 38 regencies in East Java. An illustration for Bojonegoro regency, estimated model for explaining the relationship between the maternal mortality and infant mortality case by percentage of early marriage on the child-girl are:

$$\hat{\theta}_1(x) = \exp(2.775282 + 0.053417(x - 24.13)), \quad x \in (24.13 \pm 0.67) \tag{10}$$

$$\hat{\theta}_2(x) = \exp(2.570254 + 0.13352925(x - 24.13)), \quad x \in (24.13 \pm 0.67) \tag{11}$$

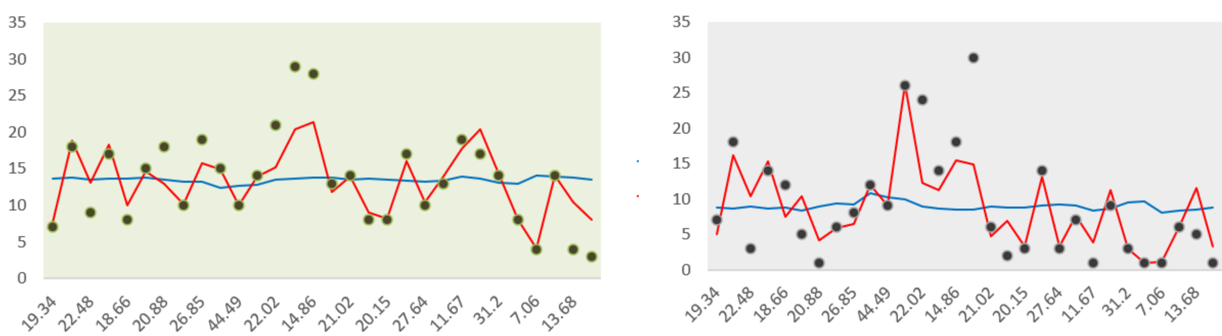
Bi-response Poisson regression model in equation (10) and equation (11) gives an estimation result when percentage of early marriage on the child-girl in is 24.13%, the average maternal mortality in Bojonegoro regency is 16.04315, the real value is 17 people and the average mortality rate for children under five is 13.06914, the real value being 14 people. It can be interpreted that for the percentage of early marriage on the child-girl in between 23.46% to 24.8%, if the percentage of early marriage on the child-girl increased by 1 percent then the average maternal mortality in Bojonegoro regency will increase by exp (0.053417) or 1 person and the average infant mortality will increase by exp (0.13352925) or 1 person.

Bi-response Poisson regression model estimate with parametric and nonparametric regression approaches based on local linear estimator for modeling effect of early marriage on the child-girl to maternal and infant mortality in East Java with the help of software (OSS)-R. The deviance statistic can be used to compare a model in parametric and nonparametric bi-response Poisson regression, a smaller deviance value indicates the best model. Deviance value between parametric and nonparametric bi-response Poisson regression approaches are given in Table 1.

TABLE 1. Deviance Value of Bi-response Poisson Regression

Goodness of fit	**Bi-response Poisson Regression**	
	Parametric approach	**Nonparametric approach**
Deviance	137.045	42.29256

Based on Table 1, the deviance value bi-response Poisson regression with a parametric regression approach is 137.045 and a nonparametric approach is 42.29256. Deviance value with the parametric approach has a better value than the nonparametric approach, so it can be concluded that bi-response Poisson regression with a nonparametric approach is better than a parametric approach. It can be seen through the comparison plot between the observation and estimation data presented in Fig. 6.



(a) (b)

FIGURE 6 . Plot Observation and Estimation Data of maternal mortality (a) and infant mortality (b).

In Fig. 6, the black dot represents the observation response variable data, the red line represents the nonparametric approach y variable estimation data, and the blue line represents the parametric approach y variable estimation data.

CONCLUSION

The scatter plot of total number of the maternal mortality and infant maternal mortality case by the percentage of early marriage on the child-girl does not pattern. Variable who doesn't have a pattern can approach with nonparametric method and for smoothing used local linear estimator. The optimum bandwidth is 0.67 and CV value is 6.381427. The optimum bandwidth used to get the best bi-response Poisson regression model. Bi-response Poisson regression model based on local linear estimator provides a good model estimate of both the maternal mortality case and infant mortality with deviance value is 42.29256, it is smaller than deviance value a parametric approach. Deviance value a parametric approach is 137.045. In bi-response Poisson regression modeling with a linear local approach, the estimated regression curves obtained are local so the results are different for 38 regencies in East Java. In addition, further research can be carried out by considering other predictor variables that are considered to have a significant.

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