Bi-response Poisson Regression Model based on Local Linear for Modelling Effect of Early Marriage on The Child-Girl to Maternal and Infant Mortality in East Java

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O U T L I N E

- Introduction
- Material
- Result
- Conclusion

INTRODUCTION

Developing countries

Early Marriage on The Child-Girl

Serious consequences for their health

INTRODUCTION

Early Marriage on The Child-Girl is defined as a marriage of a girl or boy before the age of 18 (UNICEF, 2001).

Maternal Mortality

INTRODUCTION

The death of woman while pregnant or within 42 days of termination of pregnancy, irrespective of the duration and site of the pregnancy, from any cause related to or aggravated by the pregnancy or its management but not from accidental causes.
(Dinkes Jatim, 2016)

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Infant Mortality

The death of an infant before his or her first birthday or infant under one year of age.
(Dinkes Jatim, 2016).

INTRODUCTION

Maternal Mortality & Infant Mortality

Quality of Implementation Health Service

Prosperity Level

Maternal Mortality and Infant Mortality

Indonesia : 359 deaths per 100. 000 live births & 40 deaths per 1.000 live births (BPS, 2017)

East Java 2017 : 529 deaths & 4.465 cases (Dinkes Prov Jatim, 2018)

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INTRODUCTION

Maternal Mortality (Y_1)

Infant Mortality (Y_2)

Early Marriage on The Child-Girl (X)

Bi-response Poisson regression approach based on local linear estimator

Local Linear Estimator

INTRODUCTION

Simple and easy to apply. (Liang & Zeng, 2005)

Every differentiable function can be approximated locally by a straight line (Loader, 1999)

Does not require large amounts of data for model estimation. (Nottinghom & Cook, 2001)

Fits well to boundaries. (Nottinghom & Cook, 2001)

PDF for Bivariate Poisson Distribution

INTRODUCTION

Let the random variables V_1, V_2, and U be independently Poisson distributed with parameters are θ_1, θ_2, dan γ. New random variable Y_1 and Y_2 can be constructed by: $Y_1 = V_1 + U;\ Y_2 = V_2 + U$

The probability function is

$$P(Y_1 = y_1, Y_2 = y_2) = f(y_1, y_2) = \exp\left[-(\theta_1 + \theta_2 + \gamma)\right] \sum_{k=0}^{s} \frac{\gamma^k}{k!} \frac{\theta_1^{y_1-k}}{(y_1-k)!} \frac{\theta_2^{y_2-k}}{(y_2-k)!} \quad (2)$$

where $y_1, y_2 = 0, 1, 2, \ldots$ and $s = \min(y_1, y_2)$

$$E(Y_1) = \theta_1 + \gamma \qquad Cov(Y_1, Y_2) = \gamma \qquad Corr(Y_1, Y_2) = \frac{\gamma}{\sqrt{(\theta_1 + \gamma)(\theta_2 + \gamma)}}$$

$$E(Y_2) = \theta_2 + \gamma$$

(Jung & Winkelmann,1993)

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Local Birespon Poisson Regression Model

For data points x_{ij} in neighborhood x_{0j}, we approximate $m_r(x_{ij})$ via a Taylor expansion by a polynomial of a degree 1:

$$m_r(x_{ij}) \approx \beta_{0i}^{(r)}(x_{0j}) + \beta_{1i}^{(r)}(x_{0j})(x_{ij} - x_{0j}); r = 1, 2, j = 1, 2, \cdots, p.$$

$$x_{ij} \in (x_{0j} - h, x_{0j} + h)$$

or

$$m_r(x_{ij}) = \boldsymbol{X}_r(x_0)\underset{\sim}{\beta}(x_0); r = 1, 2.$$

$$\theta_{ri} = \exp\left(\sum_{j=1}^{p} m_r(x_{ij})\right); r = 1, 2, \ i = 1, \ 2, \ \ldots, \ n, \text{and } \ j = 1, 2, \ldots, p.$$

(Darnah, 2019)

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Local Biresponse Poisson Regression Model

INTRODUCTION

Local linear estimator for $\underset{\sim}{\beta}^{(1)}(x_0), \underset{\sim}{\beta}^{(2)}(x_0),\ and\ \gamma$ in biresponse Poisson regression is the solution equation:

$$\frac{\partial L}{\partial \gamma} = \sum_{i=1}^{n} \prod_{j=1}^{p} K_{hj}(x_{ij} - x_{0j}) \left\{ 1 - \sum_{k=0}^{s} \left(\frac{k \exp\left[\boldsymbol{X}_{1i}^{T}(x_0) \underset{\sim}{\beta}^{(1)}(x_0) \right] - \gamma y_{1i}}{\gamma \exp\left[\boldsymbol{X}_{1i}^{T}(x_0) \underset{\sim}{\beta}^{(1)}(x_0) \right] - \gamma^2} + \frac{k - y_{2i}}{\exp\left[\boldsymbol{X}_{2i}^{T}(x_0) \underset{\sim}{\beta}^{(2)}(x_0) \right] - \gamma} \right) \right\} = 0$$

$$\frac{\partial L}{\partial \underset{\sim}{\beta}^{(1)}(x_0)} = \sum_{i=1}^{n} \prod_{j=1}^{p} K_{hj}(x_{ij} - x_{0j}) \left\{ -\exp\left[\boldsymbol{X}_{1i}^{T}(x_0) \underset{\sim}{\beta}^{(1)}(x_0) \right] \boldsymbol{X}_{1i}(x_0) + \sum_{k=0}^{s} \frac{(y_{1i} - k) \exp\left[\boldsymbol{X}_{1i}^{T}(x_0) \underset{\sim}{\beta}^{(1)}(x_0) \right] \boldsymbol{X}_{1i}(x_0)}{\exp\left[\boldsymbol{X}_{1i}^{T}(x_0) \underset{\sim}{\beta}^{(1)}(x_0) \right] - \gamma} \right\} = 0$$

$$\frac{\partial L}{\partial \underset{\sim}{\beta}^{(2)}(x_0)} = \sum_{i=1}^{n} \prod_{j=1}^{p} K_{hj}(x_{ij} - x_{0j}) \left\{ -\exp\left[\boldsymbol{X}_{2i}^{T}(x_0) \underset{\sim}{\beta}^{(2)}(x_0) \right] \boldsymbol{X}_{2i}(x_0) \right.$$

$$\left. + \sum_{k=0}^{s} \frac{(y_{1i} - k)(y_{2i} - k) \exp\left[\boldsymbol{X}_{2i}^{T}(x_0) \underset{\sim}{\beta}^{(2)}(x_0) \right] \boldsymbol{X}_{2i}(x_0)}{\exp\left[\boldsymbol{X}_{2i}^{T}(x_0) \underset{\sim}{\beta}^{(2)}(x_0) \right] - \gamma} \right\} = 0$$

(Darnah, 2019)

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I N T R O D U C T I O N

Kernel Function and Optimum Bandwidth Selection

$$K_h(x) = \frac{1}{h} K\left(\frac{x}{h}\right), \text{untuk} -\infty < x < \infty \text{ dan } h > 0$$

(Wasserman, 2005)

For choosing optimum bandwidth, we use cross validation (CV) criterion that has minimum value.

$$CV(h_1, h_2, \ldots, h_q) = \sum_{i=1}^{n} (y_i - \hat{y}_{-i}(x_i))^2 ; i = 1, 2, \cdots, n$$

(Li & Racina, 2004)

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Index of Distribution Test (IB)

I N T R O D U C T I O N

Index of dispersion test (IB) for bivariate Poisson distribution:

$$I_B = \frac{n(\bar{y}_2 s_{y_1}^2 - 2m_{11}^2 + \bar{y}_1 s_{y_2}^2)}{(\bar{y}_1 \bar{y}_2 - m_{11}^2)}$$

where:

$$s_{y_1}^2 = \frac{\sum_{i=1}^{n}(y_{1i} - \bar{y}_1)^2}{n} \qquad s_{y_2}^2 = \frac{\sum_{i=1}^{n}(y_{2i} - \bar{y}_2)^2}{n} \qquad m_{11} = \frac{\sum_{i=1}^{n}(y_{1i} - \bar{y}_1)(y_{2i} - \bar{y}_2)}{n}$$

Y_1 and Y_2 are bivarite Poisson distribution if:

$$I_B < \chi^2_{(\alpha, 2n-3)}$$

Best (1999)

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Correlation Coefficient

INTRODUCTION

$$r_{y_1 y_2} = \frac{\sum_{i=1}^{n}(y_{1i}-\overline{y}_1)(y_{2i}-\overline{y}_2)}{\sqrt{\sum_{i=1}^{n}(y_{1i}-\overline{y}_1)^2}\sqrt{\sum_{i=1}^{n}(y_{2i}-\overline{y}_2)^2}}$$

The test statistics is: $$t = \frac{r_{y_1 y_2}\sqrt{n-2}}{\sqrt{1-r_{y_1 y_2}^2}}$$

The null hypothesis is rejected if $|t| > t_{\left(n-2,\frac{\alpha}{2}\right)}$ (Weisberg, 2005)

If $r_{y_1 y_2} \geq \pm 0{,}5$ strong correlation.

If $r_{y_1 y_2} < \pm 0{,}5$ weak correlation.

Gogtay & Thatte (2017)

MATERIAL

The response variables are the maternal mortality and the infant mortality cases in East Java 2017 were collected from the Health Service Province (Dinkes Provinsi) East Java.

The predictor variable is the Early Marriage on The Child-Girl was collected from Central Agency on Statistics (BPS) East Java.

Once all the data was collected, we entered the data into the Open Source Software (OSS)-R for analysis.

STEP OF ANALYZE

1. Dividing of data: in sample and out sample
2. Testing bivariate Poisson distribution
3. Correlation Test for birespon variables
4. Selection optimum bandwidth

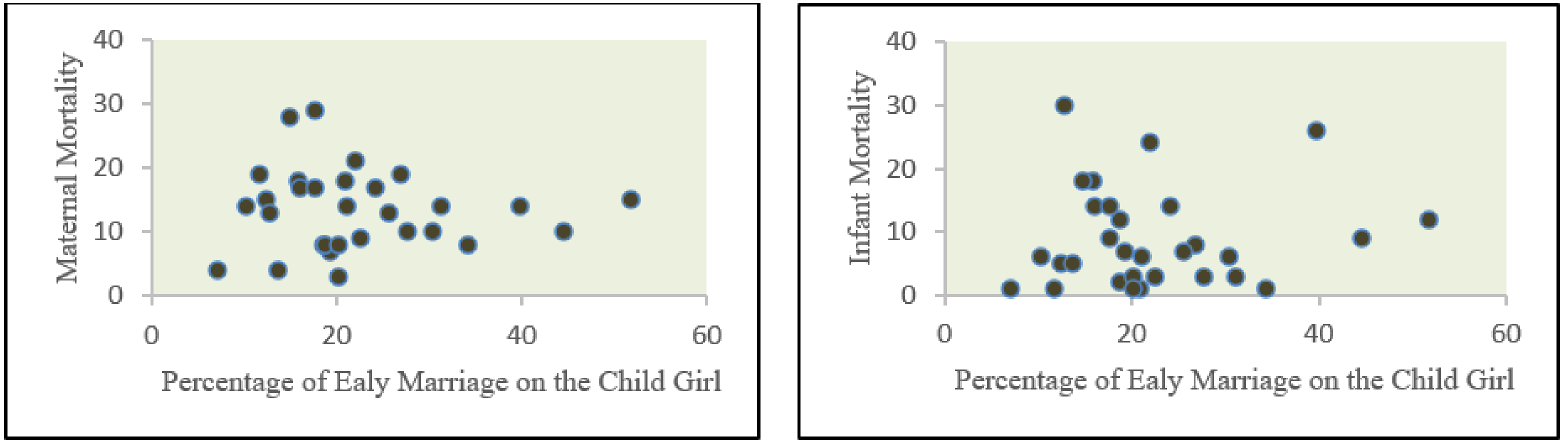
5. Modelling biresponse Poisson regression based on local linear approach
6. Modelling biresponse Poisson regression based on parametric approach
7. Comparing parametric and non parametric approach
8. Selection the best model

RESULT

Characteristics the Variables

Variable	Mean	Standard Deviation	Minimum	Maximum
Maternal Mortality	13,9	10,1	0	49
Infant Mortality	10,9	14,7	0	65
Percentage of Early Marriage on The Child-Girld	20,9	10,4	6,8	51,7

RESULT



(a) (b)

FIGURE 1. Plot of the Maternal Mortality Case (a) and Infant Mortality Case (b) by Percentage of Early Marriage on the Child Girl

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Corelation and Bi-variate Poisson Distribution Test on Response Variable

	Value	Conclusion
Index IB	-41,1	Bivarite Poisson Distribution
Chi Square	75,6	
t	2,7	Correlation is significant
t_table	2,0	
Coef.corelation	0,5	Strong correlation

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Selection Optimum Bandwidth

RESULT

Bandwidth (h)	0,65	0,66	0,67	0,68	0,69	0,70
CV	8,378115	9,212682	6,381427	9,190332	9,290435	11,06244

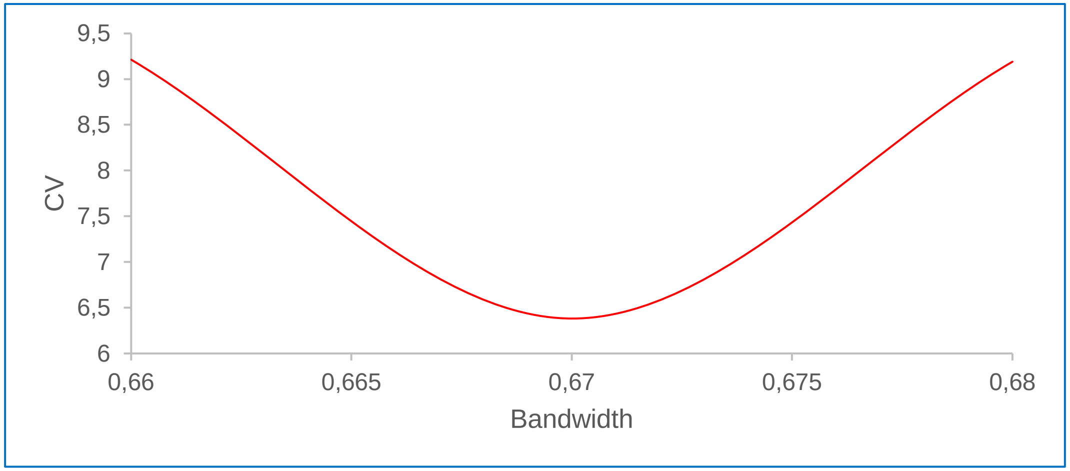

FIGURE 4. Plot of CV with bandwidth value.

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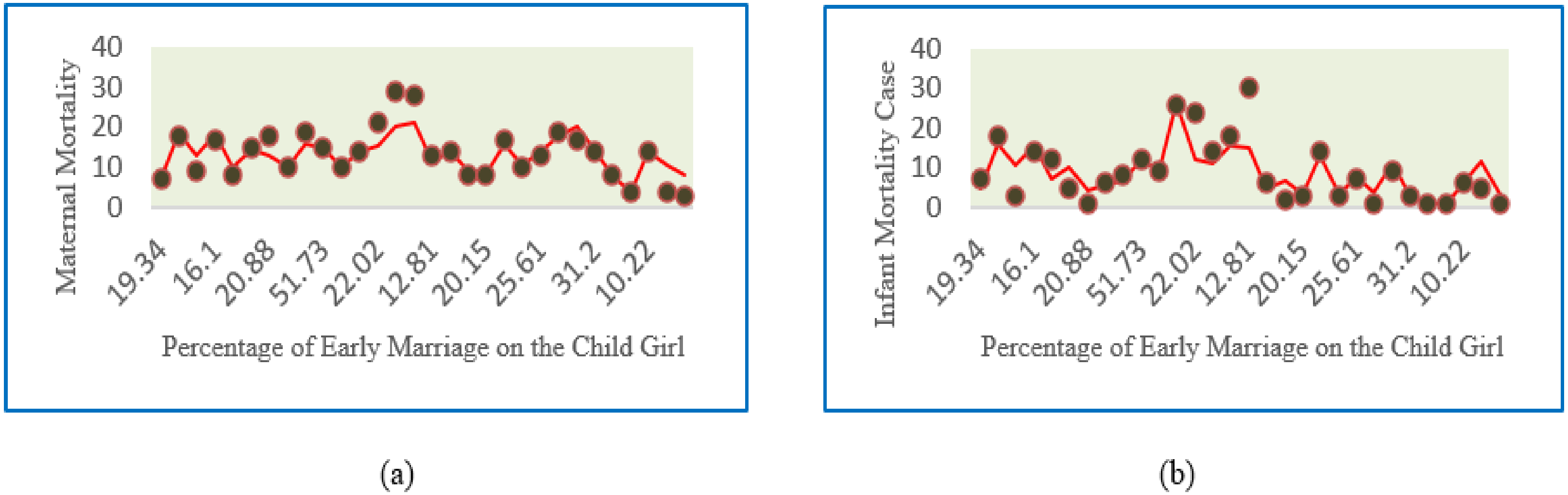

FIGURE 5. Estimation of maternal mortality (a) and infant mortality (b).

In bi-response Poisson regression modeling with a linear local approach, the estimated regression curves obtained are local so the results are different for regencies in East Java.

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Example

Bojonegoro Regency:

$$\hat{\theta}_1(x) = \exp(2.775282 + 0.053417(x - 24.13)), \quad x \in (24.13 \pm 0.67) \qquad (1)$$

$$\hat{\theta}_2(x) = \exp(2.570254 + 0.13352925(x - 24.13)), \quad x \in (24.13 \pm 0.67) \qquad (2)$$

It can be interpreted that for the percentage of early marriage on the child-girl in between 23.5% to 24.8%, if the percentage of early marriage on the child-girl increased by 1 percent then the average maternal mortality and infant mortality in Bojonegoro regency will increase 1 person.

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RESULT

Bi-response Poisson regression model based on parametric approach

$$\hat{\theta}_1(x) = \exp(2.664013 - 0.002877\,X).$$

$$\hat{\theta}_2(x) = \exp(2.050035 - 0.006337\,X).$$

Deviance Value of Bi-response Poisson Regression

Parametric
137.045

Local Linear
42.29256

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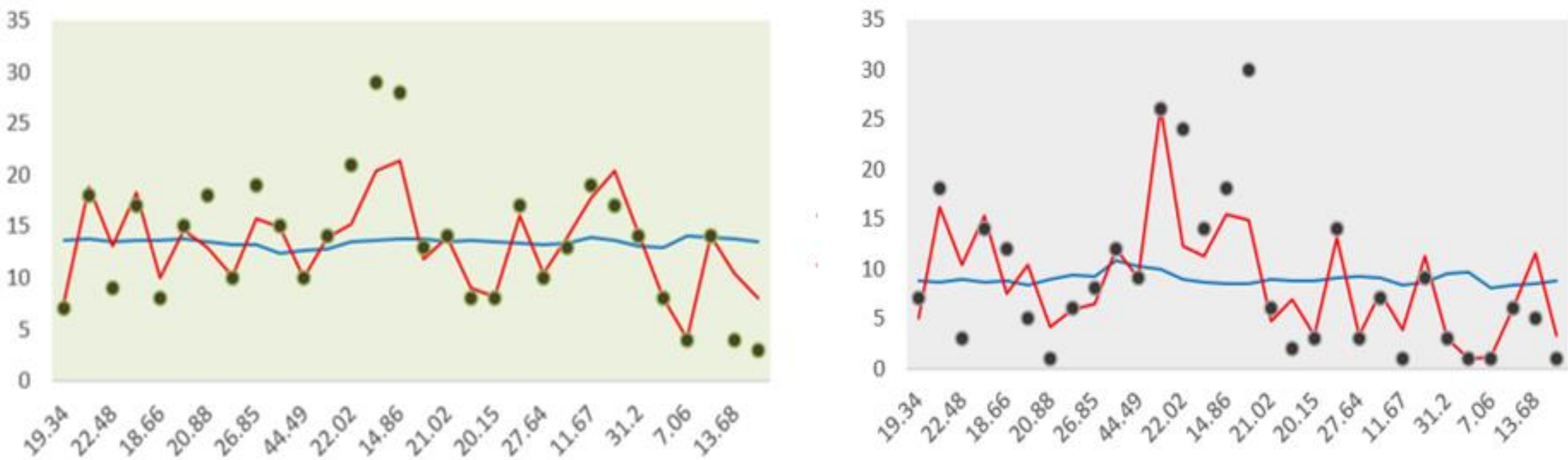
RESULT



(a) (b)

FIGURE 6 . Plot Observation and Estimation Data of maternal mortality (a) and infant mortality (b).

The black dot represents the observation response variable data, the red line represents the nonparametric approach y variable estimation data, and the blue line represents the parametric approach y variable estimation data.

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CONCLUSION

The scatter plot of total number of the maternal mortality and infant mortality case by the percentage of early marriage on the child-girl does not pattern. Variable who doesn't have a pattern can approach with nonparametric method and for smoothing used local linear estimator.

The optimum bandwidth is 0.67 and CV value is 6.381427. The optimum bandwidth used to get the best bi-response Poisson regression model.

Bi-response Poisson regression model based on local linear estimator provides a good model estimate of both the maternal mortality and infant mortality with deviance value is 42.29256, it is smaller than deviance value a parametric approach. Deviance value a parametric approach is 137.045.

In addition, further research can be carried out by considering other predictor variables that are considered to have a significant.

Thank you