PME 41

Proceedings

Singapore | July 17-22, 2017

Of the 41st Conference of the International Group for the Psychology of Mathematics Education

Volume 4

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41st PME Annual Conference
17-22 July 2017
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Cite as:
Kaur, B., Ho, W.K., Toh, T.L., & Choy, B.H. (Eds.). (2017). Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education (Vol. 4). Singapore: PME.

Website: http://math.nie.edu.sg/pme41
Proceedings are also available via http://www.igpme.org

ISBN: 978-981-11-3742-6

Cover design: Mohamed Fadzli Ibrahim, National Institute of Education, Singapore.
Printed by Welon Printing Co. Pte Ltd

TEACHERS' BELIEFS AND HOW THEY CORRELATE WITH TEACHERS' PRACTICES OF PROBLEM SOLVING

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In this report, a case study with three teachers from Indonesia is presented. The purpose of this present study is to understand if and how teachers' beliefs correlate with their practices of problem solving. The teachers were asked to teach the topic "problem solving" and the corresponding lessons were observed. Additionally, the participating teachers were interviewed to capture their beliefs regarding mathematics and problem solving. The analyses of the interviews and the observations show a correlation between the teachers' beliefs and their actions in those lessons.

INTRODUCTION

Due to the bad performances of Indonesian students in all PISA studies since 2000, The Ministry of Education and Culture of Indonesia aimed at reforming the mathematics education by publishing a new curriculum. A closer look into these documents (Safrudiannur & Rott, 2016) reveals that the changes do not only affect mathematics contents but also the process-related standards: In the new curriculum, the learning standards emphasize more on problem solving. In contrast, solving problems was only an implicit goal of the mathematics education in the old curriculum.

The reformation of the learning standards raises the issues of how Indonesian teachers implement problem solving in their teaching and, especially, how they teach to solve problems. As we know, many researchers have revealed that teachers' beliefs affect the way how they teach in their classes (Thompson, 1992; Phillips, 2007). Furthermore, Rott (2016) has shown that teachers' style in practicing problem solving often match with their beliefs of the nature of mathematics. Therefore, we are especially interested in the Indonesian teachers' beliefs about mathematics and problem solving.

To better understand how the beliefs correlate to teachers' practices, we have carried out a qualitative study by asking teachers to involve problem solving in their lessons. The underlying research question of this present study is "how do their beliefs correlate their practices of problem solving in the Indonesian educational context?"

THEORETICAL BACKGROUND

Beliefs

Beliefs play an essential role in learning and teaching of mathematics (Thompson, 1992; Philipp, 2007). Philipp (2007) defines beliefs as psychologically held

2017. In Kaur, B., Ho, W.K., Toh, T.L., & Choy, B.H. (Eds.). *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 121-128. Singapore: PME.

understandings, premises, or propositions about the world that are thought to be true. A belief does not stand isolated from the other beliefs. They are interconnected. For example, beliefs about the nature of mathematics will affect teachers' beliefs about mathematics teaching and learning.

Ernest (1989a) states that there are three beliefs regarding the nature of mathematics: the instrumentalist view, the Platonist view, and the problem solving view.

> First of all, there is the instrumentalist view that mathematics is an accumulation of facts, rules, and skills to be used in the pursuance of some external end….. Secondly, there is the Platonist view of mathematics as a static but unified body of certain knowledge…… Thirdly, there is the problem-solving view of mathematics as a dynamic, continually expanding field of human creation and invention, cultural product (Ernest, 1989a, p. 250).

Furthermore, he describes how those three views influence teachers' practices.

> For example, the instrumental view of mathematics is likely to be associated with the instructor model of teaching, and the strict following of a text or scheme. It is also likely to be associated with the child's compliant behaviour and mastery of skills. Similar links can be made between other views and models, for example: Mathematics as a Platonist unified body of knowledge – the teacher as explainer – learning as the reception of knowledge; Mathematics as problem-solving – the teacher as facilitator – learning as the active construction of understanding, possibly even autonomous problem-posing and problem-solving. (Ernest, 1989a, p. 251-252)

Ernest's theory will serve as a fundamental theoretical basis for interpreting the relations of teachers' beliefs from the interviews and the observations in this present study.

METHOD

The method of our preliminary study is observation and semi-structure interview. We asked three mathematics teachers to conduct a lesson involving one or more math problem(s). Few days before those teachers conducted the lesson, a particular problem was chosen and discussed together with the teachers, but they had options to add or pose their own problems. The lesson of each teacher was observed and filmed. The coding system from TIMSS Videotape Classroom Study 1999 was used for interpreting the underlying lessons. One of the reasons for this choice was that the TIMSS video study was conducted in several countries worldwide, including Indonesia in 2010.

We categorized the learning process of the entire lesson by applying the coding such as the classroom interactions (public, private, or mix) and the content activities (problem or non-problem segment). In this paper, we present only the problem segment.

The problem segment is a segment containing the discussion of a mathematical problem. The segment starts when a teacher states or assigns the problem. The segment ends when solutions have been found or the discussion around the solution has been finished, whichever occurs in the final stage of the segment (NCES, 1999). The segment is characterized by the problem statements, the processes of searching for a solution, discussion including the checking of the solutions, and others (transitions between them).

At the end of each lesson, we interviewed the teachers in order to acquire more information about their beliefs of mathematics and problem solving. The analyses of the interviews were discussed with two colleagues from the mathematics education department of the Mulawarman University in Samarinda, Indonesia.

This pilot study was implemented in three junior high schools in Samarinda. Three teachers (A, B, and C) voluntarily participated in this present study. They each have been mathematics teachers for more than ten years. Schools of teacher A and C are implementing the old curriculum (2006 curriculum) and school of teacher B is implementing the new curriculum (2013 curriculum).

RESULTS

Teacher A

Observation: Table 1 represents the problem discussed in the lesson of teacher A and the activities during the problem segment. The duration of his lesson was 82 minutes and 54 seconds (82:54). The problem segment lasted for 42:40 (51.5%).

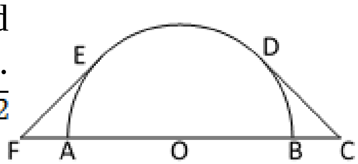
Activities	Duration
Problem Statement	08:52
The problem: O is the centre of the semi-circle. D and E are the tangent points of CD and FE respectively. $\|\angle BCD\| = \|\angle AFE\| = 45^{o}$. If the length of OC is $\sqrt{392}$ cm, find the total length of CD, DE, and EF.	
Process to find the solution	32:43
Interactions during the process in chronological order: 1. Public Interaction 1 (09:55) 2. Private Interaction 1 (01:18) 3. Public Interaction 2 (03:15) 4. Private Interaction 2 (01:36) 5. Mix interaction 1 (00:56) 6. Private Interaction 3 (00:49) 7. Public Interaction 3 (04:51) 8. Private Interaction 4 (08:27) 9. Mix interaction 2 (01:00) 10. Private Interaction 5 (00:36)	
Discussion (Checking) on the solution	00:07
Other	00:58

Table 1: Activities during the problem segment of teacher A's lesson.

Table 1 shows that in this lesson, public interactions dominates the interactions during the process to find an answer. In the public interactions, teacher A helped his students by explaining not only the step how to solve the problem, but also the necessary concepts and formulas required to solve the problem. For example, in the first public interaction, he told his students to draw lines connecting points O and D as well as O

and *E*. He guided them to recognize that *CDE* was an isosceles right triangle, and he reminded them of the appropriate formula to find the length of *CD* and *OD*.

All private interactions, in which the students worked on their own, were initiated by teacher A by instructing the students to do calculations. For example, in the first private interaction, teacher A asked the students to find the length of *CD* and *DO* by using the Pythagorean Theorem. Time of the first private interaction was limited due to increasing difficulties encountered by some students. Afterwards, teacher A continued to guide students applying the theorem in the second public interaction.

Interview: For teacher A, a problem is a task that is difficult for students to solve. A student is successful in problem solving if the student gets the correct answer. To ensure that, he guided his students step by step in his lesson. He said that without his guidance, his students would not be able to find the answer. He assessed that his students in the current lesson had low mathematical abilities.

The way he guided his students is harmonious with his beliefs about mathematics. He expressed that he somehow believes that mathematics is an accumulation of useful facts, rules, and skills. Students should know which mathematical formulas are appropriate to solve a problem and know how to apply them. Those expressions indicate that teacher A holds the *instrumentalist view*. Thompson (1984) argued that if a teacher holds this belief of mathematics, it is important that students are able to recall what the teacher taught and then apply it to obtain the correct answer.

Teacher B

Observation: Table 2 visualizes teacher B’s activities during the problem segments. The total time of his lesson was 80:58. He posed his own three problems regarding mean values. Time of the first, second, and third problem segment were 10:22 (12.8%), 13:42 (16.9%), and 21:38 (26.7%), respectively, or 45:42 in total.

Table 2 shows that private interactions and mix interactions dominate the process of obtaining a solution in all problem segments. The public interaction in second problem segment has occurred because teacher B explained the text of the second problem. He wanted to ensure that his students understood the problem.

In contrast to teacher A, he did not give any clues to his students. In each problem segment, he repeatedly encouraged his students to create their own strategies. He emphasized that his students could use their own formulas.

Interview: The problems posed by teacher B reflect what his beliefs about a mathematical problem. For teacher B, a problem should be the application of mathematics in the real world.

The way he guided his students in problem solving is also influenced by his beliefs about the nature of mathematics. He released his students to create their own formulas because he strongly disagreed that mathematics is an accumulation of facts, formulas, or skills. It is not obligatory for students to memorize formulas. For him, mathematics contents are not fixed but can change and are open for revision. From the interview, teacher B seems to hold the *problem solving view*.

Activities	Duration
First Problem Segment:	
Problem Statement	01:25
The problem: The average height of eight volleyball players is 176 cm. After two players leave, the new average is 175 cm. Find the average height of the two players!	
Process to find the solution	05:56
Interactions during the process in chronological order:	
1. Private Interaction 1 (02:42)	
2. Mix interaction 1 (03:14)	
Discussion on the solution	01:02
Other	01:59
Second Problem Segment:	
Problem Statement	01:55
The problem: The average weight of six futsal [a ball game] players is 65 kg. After a substitution, the new average weight is 63.5 kg. If the weight of the player who left is 64 kg, find the weight of the new player.	
Process to find the solution	09:15
Interactions during the process in chronological order:	
1. Public interaction 2 (01:09) 3. Mix interaction 2 (05:18)	
2. Private Interaction 2 (02:48)	
Discussion on the solution	01:37
Other	0:55
Third Problem Segment:	
Problem Statement	01:17
The problem: The math test average score of a group of students is 63. If a student whose score is 80 is included to the group, the new average score is 64. Find the initial number of the students in the group.	
Process to find the solution	14:54
Interactions during the process in chronological order:	
1. Private Interaction 3 (11:20) 2. Mix interaction 3 (03:34)	
Discussion on the solution	03:07
Other	02:20

Table 2: Activities during the problem segments of teacher B's lesson.

Teacher C

Observation: Duration of his lesson is 73:41 and the problem segment lasted for 24:38 (33.5%). Before the problem segment, teacher C announced that his students are allowed to use their own strategies but the strategies should follow mathematical rules. Then he gave them two mathematical tasks. Both tasks were based on two mathematical facts respectively: the total size of three angles forming a straight angle is 180^o and the total size of three angles of a triangle is 180^o.

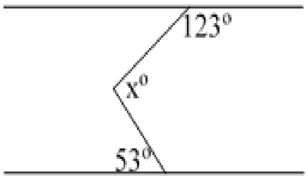
Activities	Duration
Problem Statement	00:40
The problem: Find *x*! 	
Process to find the solution	23:43
Interactions during the process in chronological order: 1. Private Interaction 1 (14:41) 2. Mix interaction 1 (07:16) 3. Private Interaction 2 (01:46)	
Discussion (Checking) on the solution	00:00
Other	00:15

Table 3: Activities during the problem segment of teacher C's lesson.

In the process to find a solution, private interactions dominated the process. During the private interactions, students worked individually and teacher C walked around and motivated them to show their works on the whiteboard. To assist his students, he gave a clue that students needed to draw an auxiliary line to solve the problem.

In the mix interaction, there were two students showing their works on the whiteboard. After they finished writing their solutions, teacher C clarified their works by asking some questions. For example, one of the two students wrote the equation $123+x+53=180$ and teacher C asked him to explain how he got it.

Interview: For teacher C, a mathematical problem is a task that is difficult for students to solve. To succeed in solving it, he said that his students needed his help. He gave clues consisting of concepts or ideas related to the problem, not how to solve it.

For him, concepts are crucial for solving problems successfully. He said that the two tasks before the problem segments were his clues since the concepts in the two tasks would be useful and related to the problem. In addition to the two task, he also gave the students an idea by telling them to draw an auxiliary line which could help them to apply the concepts. He said that if there was enough time, he would draw it.

He believes that mathematics contents are not fix but dynamics, can change over time, and are open for revision. He strongly disagrees that students should memorize formulas. Thus, he does not care how his students solve a problem as long as the approaches follow mathematical rules. From this interview, teacher C seems to hold the *problem solving view*.

DISCUSSION

The results show that the observed teachers' beliefs of the nature of mathematics correlate with the way in which they involve problem solving in their lessons. We interpret this in the following way: the beliefs influence the teaching style.

Teacher B who holds the *problem solving view* does not require his students to memorize and apply formulas that he taught. He encouraged his students to create their own strategies or even their own formulas to solve problems. The coding of his lesson shows that private and mix interactions dominate the process to get an answer. In the private interactions, students work privately and he encouraged them to create their own formulas. In the mix interactions, one or two students show their works and other students can look at the works or still work on problems. These interactions indicate that he gave his students a lot of time to work. He did not disturb his students by giving clues what formulas are appropriate. He gave his students opportunities to work on their own.

Contrastingly, teacher A shows a different style on teaching problem solving since he has a different view. He holds the *instrumentalist view* which mathematical formulas are very important. His view influences how he guides his students on problem solving. He tried to direct his students to solve the problem by reminding them of appropriate formulas and concepts and also guiding them how to apply the formulas or concepts. He did not encourage students to create their own strategies. He also did not introduce alternative ways to solve the problem. He believed that it would confuse his students. Apparently, it is enough for his students to copy his procedures. In the mix interaction, students' products on the whiteboard were the applications of his clues.

Teacher A and B's actions show that their beliefs about the nature of mathematics and their style in teaching problem solving are related to each other. But, compared to teacher A, there is a gap between the way teacher C guided his students in problem solving and what he believes about mathematics.

The interview indicates that teacher C holds the *problem solving view*. At the beginning of his lesson, he told his students that they can develop their own strategies to solve problems. But during the process to find the answer, he did not encourage them to express their own ideas. He gave clues which could lead his students to apply his clues. This can be seen from the example of a student who wrote his answer during mix interaction. This student tried to apply teacher C's clues but did so incorrectly. He wrote down the equation $123+x+53=180$. With this equation, he tried to follow the equations to find the answers of the two tasks that were posed before the current problem segment. In those two tasks, the equations needed to find the answers used the fact that the total size of three marked angles was equal to 180.

Teacher C insisted that he wanted to free and encourage his students to create their own strategies but he was sure that they could not do that. He gave clues due to the low mathematical performance of his students, which led to the assumption that his students would not be able to develop strategies for problem solving.

CONCLUDING REMARKS

This present study shows that in the Indonesian educational context, teachers' beliefs about the nature of mathematics correlate with their practices of problem solving. What they believe about mathematics can be matched with their style in teaching problem solving. However, there are factors which can make a gap between teachers' beliefs about the nature of mathematics and their way to teach problem solving in their lesson. One of them is students' low abilities in mathematics.

In the next study, we are going to develop a quantitative instrument to collect beliefs of a large sample of Indonesian teachers. This present study has shown that teachers' beliefs of the nature of mathematics are an important factor to understand how teachers practice problem solving. Therefore, the theory about teachers' beliefs about the nature of mathematics will serve as basis to develop the instrument.

We also consider counting on students' mathematics abilities in our instrument. This study has found that students' mathematical abilities can explain a gap between teachers' beliefs and their observed actions. We suppose that students' mathematical abilities can be one of the social contexts which according to Ernest (1989a) can cause disparities between a teachers' espoused and enacted model of teaching mathematics. By considering students' mathematical abilities, we can better understand the disparity between teachers' beliefs and their actual teaching especially in problem solving.

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